Performances of a multi-static model of sound scattering by rough surfaces

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The ocean floor is far from being a smooth and perfectly rigid surface. That is why its sound scattering properties are a useful input to the analysis of this medium as for acoustic data inversion. Thus, scattering strength has been investigated at high frequency of order of 10kHz to hundreds of kHz under different geometrical configurations. The Kirchhoff Approximation and the Small Perturbation Method could be cited respectively in the case of dimensions of a rough surface larger and smaller than the wavelength. Nevertheless, modeling a rough surface should be considered at different scales compared to the wavelength. As a part of the incident wave is transmitted to the seabed medium, it is also important to know the effect of the scattering coming from the volume. Jackson’s scattering model takes these considerations into account. The aim of our study is first to show an improvement of the surface scattering method using the Small Slope Approximation and keeping the initial method of Jackson’s model to describe the scattering from the volume. Comparisons with a well-known model are presented to show the performances of this new approach and comparisons between different geometries are analyzed to show the most useful configurations of the model.

1 Introduction

Acoustic scattering from ocean bottom is a subject of interest to the underwater acoustic community. Different studies have been carried out considering the complexity of the ocean bottom. This complex acoustic medium may be assumed as elastic-solid, fluid or porous to solve scattering problem. These assumptions make the models have different limitations. In this paper, the models deal with sound waves at high frequencies for a scattering fluid(water)-fluid(seafloor) interface with a rough surface between both media. The roughness scattering is predicted with the Small Slope Approximation (SSA) which was first developed by Voronovich [8]. The inhomogeneities in the sediment are analyzed as volume scattering with a model first formulated by Mourad and Jackson [7] for a backscattering configuration and then generalized for the bi-static case [2]. A full process is described as the sum of both components, the roughness scattering and the volume scattering. In particular cases, the roughness scattering level is higher than the volume scattering, thus the total scattering is said to result from the roughness of the surface. For other situations, the volume scattering effect is greater than the roughness effect or may be more or less similar to the roughness scattering component. These different situations depend on the media, the frequency of the emitted wave, the angles of the incident and scattered waves, and so on. We are more interested by the roughness scattering even if volume scattering must be included in the scattering implementation to reach a valuable model. For the sake of simplicity, this full process is called ”SSA-volume” and is the sum of roughness and volume scattering strength components.

In this paper, the Small Slope Approximation of first order is shown to be pertinent enough to replace the roughness scattering theory used in the bi-static model developed in [2, 4] and called ”Jackson’s model”. The volume scattering component of SSA-volume model is similar to the one implemented in Jackson’s model. In Jackson’s model, roughness scattering strength is predicted by an interpolation of the Kirchhoff Approximation (KA) and the Small Perturbation Method (SPM). KA is used to evaluate scattering strength in the specular directions, whereas SPM is used for predictions in other directions than the specular one and works well with a roughness relief smaller than the acoustic wave length. Small Slope Approximation has been elaborated as an unifying method that could reconcile SPM and KA without separating the spectrum of the rough surface into large and small deviations of the relief. The validity condition of this method consists only in the smallness of the elevation slopes, without any restriction on the sound wavelength which is a limitation for the other scattering models.

In Section 2, the scattering problem is described in terms of incident and scattered waves. The environment where the process takes place plays a crucial role in the scattering analysis and is briefly presented as well as the theory of the SSA-volume model. Then a simulation study is carried out in Section 3. Scattering strength is expressed for the scattering case in one plane which is a special case of the bi-static configuration. Next, the azimuth angle of the scattered angles is changed to get a global view over the configuration. The scattering predictions are discussed in Section 4.

The use of SSA for predicting roughness scattering works well and perspectives about this method are clear as the possible use of it with a more complicated rough interface. Finally, the aim of this study is first to show that SSA performs as well as the roughness predictions of Jackson’s model under similar configurations. Then, SSA can be suitable to many fluid environments with more complex roughness than the one used in this paper. This would be out of the limits where Jackson’s model is applicable. Those predictions should be presented in a future paper under its full process, SSA-volume, to reach as closer as possible the reality of different seabeds.

2 Background

The geometry of the scattering problem is depicted in Figure 1 in terms of incident and scattered sound waves. $k_i$ and $k_s$ represent respectively the incident and scat-

![Figure 1: Geometry of the scattering problem.](image-url)
tered wave vectors.

\[ k_i = \{K_i, -k_{zi}\}, \quad k_s = \{K_s, k_{zs}\} \]  

(1)

where \( K_i \), \( K_s \) are the transverse components of the incident and scattered waves in the \((x, y)\) directions, so \( k_i = \{k_{zi}, k_{yi}\} \) and \( k_s = \{k_{zs}, k_{ys}\} \). The vertical components in the \(z\)-direction, \( k_{zi} \) and \( k_{zs} \), respect \( k_z = \sqrt{k^2 - (k_x^2 + k_y^2)} \), with \( k \) the wavenumber. Notice that the scattering theory depends on the grazing angles \( \theta_i \), \( \theta_s \) and the azimuth angles \( \phi_i \) and \( \phi_s \). The scattering process depends also on the parameters of the media where sound scattering takes place. Medium 1 is the water. Its sound velocity is assumed to be constant with \( c_1 = 1500 \text{m/s} \) and to have a mass density equal to \( \rho_1 = 1000 \text{kg/m}^3 \). Medium 2 is described by many parameters related to the roughness and the inhomogeneities of the seafloor. This set of data is found in Table 1. This has been established for different seafloors and they are found in [3, 4, 5].

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>Density ratio ((\rho_2/\rho_1))</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Sound speed ratio ((c_2/c_1))</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Loss parameter</td>
</tr>
<tr>
<td>( w_2 )</td>
<td>Roughness spectral strength</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>Rough spectral exponent</td>
</tr>
<tr>
<td>( w_3 )</td>
<td>Volume spectral strength</td>
</tr>
<tr>
<td>( \gamma_3 )</td>
<td>Volume spectral exponent</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Fluctuation ratio</td>
</tr>
</tbody>
</table>

Table 1: Seafloor parameters

### 2.2 Modeling acoustic scattering: a brief outline of theory

The scattering strength, \( S \), predicted in Section 3 is defined in decibel (dB) as:

\[
S(\theta_i, \phi_i, \theta_s, \phi_s) = 10 \log_{10}[\sigma_r(\theta_i, \phi_i, \theta_s, \phi_s) + \sigma_e(\theta_i, \phi_i, \theta_s, \phi_s)]
\]  

(7)

where \( \sigma_r(\theta_i, \phi_i, \theta_s, \phi_s) \) is the roughness contribution and \( \sigma_e(\theta_i, \phi_i, \theta_s, \phi_s) \) the volume contribution to the scattering cross section per unit area. \( \sigma_r(\theta_i, \phi_i, \theta_s, \phi_s) \) is evaluated with the Small Slope Approximation at first order and is expressed as:

\[
\sigma_r(\theta_i, \phi_i, \theta_s, \phi_s) = \frac{|A_{spm}(\theta_i, \phi_i, \theta_s, \phi_s)|^2}{2\pi(k_{zi} + k_{zs})^2} \times \int_0^{\infty} \left( e^{-\{[(k_{zi} + k_{zs})D(0)] - e^{-[(k_{zi} + k_{zs})D(\infty)]}) \right) J_0(Qr)dr
\]  

(8)

where \( D \) is the structure function, \( k_{zi}, k_{zs}, K_i \) and \( K_s \) are the wave vector components defined before in Section 2. Notice that the rough surface is considered as isotropic and this allowed the integral of Eq(8) to be simplified. The roughness scattering component is finally:

\[
\sigma_r(\theta_i, \phi_i, \theta_s, \phi_s) = \frac{|A_{spm}(\theta_i, \phi_i, \theta_s, \phi_s)|^2}{2\pi(k_{zi} + k_{zs})^2} \times \int_0^{\infty} \left( e^{-\{[(k_{zi} + k_{zs})D(0)] - e^{-[(k_{zi} + k_{zs})D(\infty)]}) \right) J_0(Qr)dr
\]  

(9)

where \( J_0 \) is the zeroth-order Bessel function of the first kind, \( Q \) is the magnitude of the transverse component difference modified [5] to prevent numerical difficulties as:

\[
Q = \sqrt{(k_{zi} - k_{zs})^2 + (k_{ys} - k_{ys})^2 + (k \times 0.001)^2}
\]  

(10)

\( A_{spm} \) is a coefficient which depends on the properties of the lower medium and is obtained using first-order perturbation theory for the corresponding problem. For a fluid-fluid multi-static problem, this component is defined as:

\[
|A_{spm}(\theta_i, \phi_i, \theta_s, \phi_s)|^2 = \frac{k^4}{T} \times \left| \Gamma(\theta_s) + 1 \right| \left| \Gamma(\theta_i) + 1 \right|^2 \times \left( 1 + \frac{\sigma_s^2}{\sigma_i} + \left( \frac{1}{\rho} - 1 \right)^2 \left( \frac{K_i K_s}{k^2} - \frac{P_s^2(\theta, \phi)}{\rho} \right)^2 \right)
\]  

(11)
where \( \rho = \rho_2/\rho_1, \kappa = k_2/k_1 = c_1/c_2, k = 2\pi f/c_1 \) with \( f \) the frequency of the incident plane wave. \( \Gamma(\theta) \) is the plane-wave reflection coefficient and \( P_g^2 \) is equivalent to the expression [6]:

\[
P_g^2(\theta_s, \theta_s) = \rho^2 \sin(\theta_s) \sin(\theta_i) \times \left( \frac{1 - \Gamma(\theta_s)}{1 + \Gamma(\theta_s)} \right) \left( \frac{1 - \Gamma(\theta_i)}{1 + \Gamma(\theta_i)} \right)
\]

Once the roughness contribution is established, the volume scattering is briefly described and corresponds to the component used in [2, 4]:

\[
\sigma_v(\theta_i, \phi_i, \phi_s, \phi_s) = \left[ \frac{1 + \Gamma(\theta_i)^2}{2k\rho^2} \left( \frac{\rho_1}{\rho_2} + \frac{\rho_2}{\rho_1} \right) \sigma_{pm} \right] \mu \kappa
\]

\( \sigma_{pm} \) is the coefficient obtained by the perturbation method and adapted by [2] as:

\[
\sigma_{pm} = \frac{\pi k^4 \mu \kappa^2 + \cos(\theta_s) \cos(\phi_s - \phi_i)}{2P(\theta_i)P(\theta_s)} W_{\rho\rho}(\Delta k)
\]

\( W_{\rho\rho}(\Delta k) \) is the spectrum for density fluctuation,

\[
W_{\rho\rho}(\Delta k) = \frac{w_3(\Delta k)}{(\Delta k)^3}
\]

evaluated at the Bragg wave number, thus \( \Delta k \) is the magnitude difference between the real part of the incident and the scattered three dimensional wave vectors.

\[
\Delta k = k|4Q^2 - (Re[P(\theta_i) + P(\theta_s)])|^2|^{1/2}
\]

Note that acoustic loss is taken into account in the volume scattering coefficient by allowing the wave number in the seabed to be complex. \( P(\theta) = \sqrt{\kappa^2 - \cos(\theta)^2} \) is the complex wave number in the sediment divided by the real wave number in the water.

The main components have been described for simulating SSA-volume model. The comparison with the well-known Jackson’s model has been done thanks to the expressions coming mainly from [4].

### 3 A simulation study

A simulation study has been carried out with different configurations. Scattering strength predictions are made from SSA-volume. Four types of seafloor are considered in this paper which are silt, medium sand, coarse sand and sandy gravel. Parameters describing those floors are given in Table 2. These different kinds of seafloor make the relief have many dimensions, different sound absorption effects, and so on. Notice that those media can be assumed as fluid media where only the compressional wave is of importance. This would not be possible with a rock medium where shear waves are also very noticeable compared to compressional waves. All plots represent the scattering strength predicted from SSA-volume model compared to Jackson’s model, except Figure 3 which shows the prediction of scattering strength from SSA-volume model, the prediction of roughness scattering from SSA and the volume scattering effect. The purpose of Figure 3 is to see whether our study of roughness scattering is pertinent in the global scattering process.

<table>
<thead>
<tr>
<th>Bottom</th>
<th>Types of seafloor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sandy gravel</td>
</tr>
<tr>
<td>( \rho )</td>
<td>2.492</td>
</tr>
<tr>
<td>( \mu )</td>
<td>1.337</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.01705</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>3</td>
</tr>
<tr>
<td>( w_2 )</td>
<td>0.000118</td>
</tr>
<tr>
<td>( \theta_3 )</td>
<td>3</td>
</tr>
<tr>
<td>( w_3 )</td>
<td>0.000377</td>
</tr>
</tbody>
</table>

Table 2: Properties of many seafloors (from [5])

### 3.1 Scattering in one plane

In the first test case, the seafloor is made of sandy gravel and has the properties described in Table 2. The incident plane wave has a frequency of 30kHz, a grazing angle \( \theta_i = 10^\circ \) first. Next \( \theta_i \) is set to 40\(^\circ\). The geometrical configuration is in one plane, thus \( \phi_s = \phi_i = 0^\circ \). Figure 2 shows the predicted scattering strength as function of \( \theta_s \).

![Figure 2: Scattering strength as function of \( \theta_s \), for \( \theta_i = 10^\circ \) and \( \theta_i = 40^\circ \), \( \phi_i = \phi_s = 0^\circ \), \( f=30kHz \), with a sandy gravel seafloor.](image-url)
Figure 3: Effects of volume and roughness scattering as function of θs, for θi = 10°, φi = φs = 0°, f=30kHz, top:silt; bottom:sandy gravel.

Figure 4: Scattering strength as function of θs, for θi = 10°, φi = φs = 0°, f=30kHz, top:silt; bottom:sandy gravel.

Figure 5: Scattering strength as function of θs, for θi = 40°, φi = 0°, f=10kHz and f=100kHz, with a coarse sand as bottom.

Figure 6: Scattering strength as function of φs, for θi = θs = 40°, φi = 0°, f=30kHz, top: sandy gravel; bottom: medium sand.

Figure 7: Scattering strength as function of φs, for θi = θs = 40°, φi = 0°, f=10kHz and f=200kHz, with a coarse sand as bottom.

3.2 Sound source and receiver in different planes

Simulations are performed with configurations where the azimuth scattered angle φs changes. Figure 5 shows the scattering strength in a case of an incident plane wave at 30kHz emitted with a set of angles θi = 40° and φi = 0°. The scattering is predicted with an azimuth angle φs = 45° first (top of Figure 5) and then with φs = 135° (bottom of Figure 5). Scattering strength is shown as function of the scattered angle θs. Two types of seafloor are considered, a sandy gravel medium and a coarse sand medium. The scattering strength predictions of SSA-volume model are similar for the different seafloor and for the two azimuth scattered angles that have been simulated compared to Jackson’s model. The maximum energy observed with a clear peak in the previous cases is not seen here in Figure 5.

Figure 6 shows scattering strength as function of the scattered azimuth angle φs. The incident plane wave is set to θi = 15° and φi = 0°. The scattered grazing angle is equal to the incident grazing angle, i.e. θs = 15°. Those angles make the plane waves be close to the horizontal plane. The results are predicted with sandy gravel first and then with medium sand. The prediction with both models are still similar, except for the case with a medium sand seafloor. A slight difference, less than 2dB, is seen in the backward direction between φs = 100° and φs = 180°. Strength at φs = 0 represents the energy in the specular direction.
Figure 7: Scattering strength for a medium sand seafloor as function of φs, for θi = θs = 40°, φi = 0°, top: f=10kHz; bottom: f=200kHz.

one for the configuration with f=200kHz (bottom of Figure 7). At different frequencies, a surface is more or less rough. Higher the frequency is, rougher a sandy seafloor is. That is why the energy in the specular direction is lower for higher frequencies. Notice that the volume scattering effect is less and less noticeable when the frequency increases. Low frequency waves penetrate into a medium more than high frequency waves for a given seafloor. Thus plane waves with higher frequencies are less affected by volume scattering.

4 Discussion

SSA-volume model predicts basically the same scattering strength as the Jackson’s model does in its limits of predictions. SSA is an unified method, whereas the roughness component of Jackson’s model is the interpolation of both methods KA and SPM. Scattering predictions with SSA in the specular direction are well done as the predictions in the scattered direction, without restriction on grazing angles. SPM is used for predicting away from the specular direction and is limited to small surface roughness, whereas SSA is not restricted to this type of surface. There is no need of a cross section between a method for prediction in the specular direction and another in the scattered direction. Models which are based on the connection between two models as Jackson’s model, can meet limitations if both KA and SPM data does not cross each other during their connection. Another point should be mentioned. Jackson’s model is used here under a multi-static form. Its backscattering version exists and is based on KA and the Two Scale Model. Despite the fact that this version has not been written with a multi-static geometry, this takes multi-roughness into account as SSA does. Furthermore, SSA works well for many configurations and this would be of importance when the surface is not isotropic. The few differences (less than 2 dB) seen between predictions of SSA-volume and Jackson’s model correspond to angles where KA and SPM meet and at extreme grazing angle in certain cases. Notice that the integral of SSA has been numerically solved and the way to solve it may be more or less complicated depending on the scattering problem. SSA-volume model performed well for isotropic surface with different dimensions and is intended to perform for anisotropic sandy surface as sandy ripples which got directional spectrum. Our simulations confirm that the use of SSA is of interest only if the roughness scattering strength is greater than the volume scattering. Predictions of scattering for a seafloor going from fine sand to sandy gravel seem to be relevant since the scattering volume is still less important than the roughness scattering. Moreover the rougher surface that has been studied still makes the sound waves be only compressional and shear waves are avoided.

5 Conclusion

Predictions of roughness scattering can be based on Small Slope Approximation, combined to a volume scattering prediction model to cope with existing scattering problems. In this paper, an isotropic interface is assumed and a simplified expression of SSA results from this surface behavior. The change of such surface by one with a directional spectrum would make the numerical integral in SSA more complicated to solve but the predictions would be of importance. Another perspective would be to predict roughness instead of roughness scattering via the structure function by extraction of this component from the expression of SSA.

References