Prediction of the Excitation Force Based on the Dynamic Analysis for Flexible Model of a Powertrain

Y.S. Kim, S.J. Kim, M.G. Song and S.K. Lee

Inha University, Mechanical Engineering, 253 Yonghyun Dong, 402-751 Inchon, Republic of Korea
sangkwon@inha.ac.kr
The powertrain is one of the important sources for the interior noise. In order to predict the interior noise due to a powertrain, the experimental method has been used based on the TPA (transfer path analysis). Although this experimental method is a useful tool for the identification of the noise source and the transfer path due to the powertrain, it is difficult to modify the structure of a powertrain by using the experimental method for the reduction of vibration and noise. In order to solve this problem, the paper presents the novel approach for the prediction of interior noise caused by the vibration of the powertrain based on the hybrid TPA technology. Therefore, the vibration of the powertrain in a vehicle is numerically analyzed by using FEM (finite element method). The vibration of the other part in a vehicle is investigated by using the experimental method based on VATF (vibro-acoustic transfer function) analysis. These two methods are combined for the prediction of interior noise caused by powertrain. This paper presents the prediction of the excitation force of the powertrain to the vehicle body based on numerical simulation.

1 Introduction

In response to consumers’ demand for a comfortable vehicle, reducing noise and vibration of the vehicle is becoming more important task in automotive engineering. Therefore, CAE (computer aided engineering) technology based on numerical simulation that make it possible to reduce total time schedule for a vehicle development and to predict the NVH (noise, vibration and harshness) performance has been widely applied to the NVH development of the vehicle [1-3]. The aim of this research is to develop a simulation method which can predict the interior noise caused by the powertrain vibration. The powertrain vibration generates interior noise in two paths: air-borne and structure-borne. The former is an acoustic transfer path of the powertrain noise radiation. The latter is a transfer path of the powertrain vibration. The interior noise by structure-borne is a complex problem because this noise related to the car body and powertrain, mount brackets, etc. Therefore, it is difficult to accurately predict the interior noise by structure-borne path based only on the numerical method [4-6]. The traditional TPA (transfer path analysis) has been used for the prediction of interior noise by the structure-borne path based on experimental method [5]. However, it is hard to modify the powertrain component for the reduction of interior noise using only experimental method. Recently, in order to overcome this deadlock, the hybrid CAE method has been employed to the sub-frame design of the front suspension of a passenger car [7]. In this paper, hybrid TPA is developed which involves CAE technology and experimental TPA in order to predict interior noise of the powertrain vibration due to powertrain vibration. Using powertrain vibration analysis based on CAE technology, the excitation forces at powertrain mounting points can be simulated. These excitation forces are the input force of a car body. The car body is excited to vibrate and to radiate the interior noise by these forces. In order to reduce the interior noise, the forces can be changed by structural modification of the powertrain based on CAE technology. The traditional TPA predicts the interior noise by multiplying the input forces obtained from experiment together the VATF (vibro-acoustic transfer function) of a test body vehicle. The VATF gives information about coupling characteristics between the vibration of a car body and the cavity acoustics inside of a test vehicle. In this research, the hybrid TPA predicts interior noise by using simulated excitation forces instead of experimental input forces. This paper presents a systematic approach about the of the excitation force at powertrain mounting points. The CAE technology for the simulation is based on the FE (finite element) model and multibody dynamic analysis of the powertrain. Fig.1 shows a flow chart of the systematic approach for prediction of excitation forces at powertrain mounting points.

2 The baseline test for powertrain

Two ways of tests are performed to get the vibration characteristics of a test powertrain. One is the EMA (experimental modal analysis) for the powertrain in order to identify the modal parameters which are used for the validation of the FE model. The other is the driving test when the test powertrain is loaded with a vehicle. These vibration test results are used to confirm the numerical simulation for powertrain vibration based on CAE technology. The test powertrain consists of an in-line 4 cylinder diesel engine and 5-step automatic transmission which engine displacement is 2.2 liters. This powertrain is loaded on SUV (sport utility vehicle) and the engine speed is driven from 1000 rpm to 4000 rpm during the driving test.

2.1 Experimental modal analysis

EMA of the test powertrain is performed with a rig system as shown in Fig. 2. The powertrain is excited by an electronic shaker with stiff stinger and its vibration is measured at 80 points by accelerometers attached on the powertrain. The test is performed at free-free condition. Also, the test is performed with rig system for the sake of EMA for assembled powertrain which includes engine block, crankshaft, bed plate, oil pan, transmission, etc. In this case, the components are excited by an instrumented impact hammer and its accelerations are measured by accelerometers. The transfer function for all points and
modal parameters like natural frequency, mode shape and damping are calculated through the tests.

Fig.2 Setup for experimental modal analysis of powertrain components and its assembly

2.2 Vibration measurement of powertrain

The measured excitation forces at powertrain mounting points are required for validation of the predicted excitation forces by numerical simulation. However, it is difficult to measure the excitation forces at mounting points directly. Instead, 3-axis accelerations are measured at four mounting points (engine mount, front roll mount, rear roll mount and transmission mount) in this paper. Fig.3 shows accelerometers attached to each mount bracket of the powertrain to measure the acceleration.

Fig.3 Setup of accelerometers at each mount bracket

3 Finite element method of powertrain

3.1 Geometry modeling of powertrain

The FE model of powertrain is required for the analysis of vibration characteristics of the test powertrain based on numerical method. And the FE model should be validated by the measured results through EMA. The FE model also used to create a model for dynamic analysis which performs the simulation to predict excitation forces at four mounting points.

3.2 Finite element model of powertrain

The mesh production is performed for finite element analysis using the geometry model of powertrain. The powertrain is composed of piston, connecting rod, crankshaft, cylinder block, cylinder head, bed plate, oil pan, transmission case, transmission gear and mount brackets. Most of components are connected by RBE2 (rigid body element 2). The transmission gear is modelled as rigid body which has mass and inertia moment. And all bearing parts are connected with spring and damping model. Fig.4 shows the geometry model and the FE model of powertrain used for this research.

Fig.4 Geometry and FE model of the test powertrain

Finally, this FE model is confirmed with the results of EMA. The number of elements and element type for of the FE models are listed in Table 1.

<table>
<thead>
<tr>
<th>Component</th>
<th>No. of nodes</th>
<th>No. of elements</th>
<th>Element Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine block</td>
<td>41860</td>
<td>129772</td>
<td>Tetra4, RBE2</td>
</tr>
<tr>
<td>Bed plate</td>
<td>12540</td>
<td>49411</td>
<td>Tetra4, RBE2</td>
</tr>
<tr>
<td>Crankshaft</td>
<td>5931</td>
<td>22931</td>
<td>Tetra4, RBE2</td>
</tr>
<tr>
<td>Oil pan</td>
<td>2857</td>
<td>954</td>
<td>Hexa, RBE2</td>
</tr>
<tr>
<td>Transmission</td>
<td>54356</td>
<td>195825</td>
<td>Tetra4, RBE2</td>
</tr>
<tr>
<td>Assembled</td>
<td>144161</td>
<td>536912</td>
<td>Tetra4, Hexa, RBE2</td>
</tr>
</tbody>
</table>

Table 1 Element specification of powertrain FE model

Through the finite element analysis, modal parameter such as mode shape, natural frequency and modal vector are calculated. These results are compared with those of EMA to confirm the accuracy of FE model. MAC (modal assurance criterion) analysis is generally used for this validation [8]. MAC analysis is a method of evaluating the orthogonal property for natural vectors of the vibration system. If two natural vectors are the same direction vector, the MAC value becomes one. If two natural vectors have orthogonal property, the MAC value is zero. The mathematical expression is given by

\[
MAC_{ij} = \frac{\left| \Phi_{\text{TEST}}^j \cdot \Phi_{\text{FEM}}^i \right|^2}{\left( \Phi_{\text{TEST}}^j \cdot \Phi_{\text{TEST}}^j \right) \left( \Phi_{\text{FEM}}^i \cdot \Phi_{\text{FEM}}^i \right)^{\frac{1}{2}}}
\]

(1)

where \( \Phi \) is the natural vector for the natural mode shape at each natural frequency of the vibration system. The modal analysis results obtained by using FEM (finite element
method) and EMA of the powertrain are listed in Table 2. According to these results the FE model of powertrain is sufficient for vibration system because the MAC values for each mode are over 0.7 and errors are under 10%.

<table>
<thead>
<tr>
<th>Component</th>
<th>Mode</th>
<th>Shape</th>
<th>EMA (Hz)</th>
<th>FEA (Hz)</th>
<th>MAC value</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine block</td>
<td>1</td>
<td>627.8</td>
<td>629.5</td>
<td>0.94</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1045.7</td>
<td>1015.1</td>
<td>0.92</td>
<td>2.93</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1377.0</td>
<td>1361.6</td>
<td>0.76</td>
<td>1.12</td>
<td></td>
</tr>
<tr>
<td>Bed plate</td>
<td>1</td>
<td>316.3</td>
<td>332.6</td>
<td>0.95</td>
<td>5.15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>625.6</td>
<td>621.1</td>
<td>0.91</td>
<td>7.22</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>758.6</td>
<td>705.7</td>
<td>0.89</td>
<td>6.97</td>
<td></td>
</tr>
<tr>
<td>Crankshaft</td>
<td>1</td>
<td>310.4</td>
<td>311.8</td>
<td>0.86</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>432.3</td>
<td>421.5</td>
<td>0.73</td>
<td>2.50</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>724.0</td>
<td>709.3</td>
<td>0.70</td>
<td>2.03</td>
<td></td>
</tr>
<tr>
<td>Oil pan</td>
<td>1</td>
<td>183.7</td>
<td>194.6</td>
<td>0.92</td>
<td>5.93</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>488.6</td>
<td>489.1</td>
<td>0.90</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>722.9</td>
<td>766.7</td>
<td>0.80</td>
<td>6.06</td>
<td></td>
</tr>
<tr>
<td>Transmission</td>
<td>1</td>
<td>865.8</td>
<td>846.7</td>
<td>0.90</td>
<td>2.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>943.4</td>
<td>929.4</td>
<td>0.86</td>
<td>1.48</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1006.5</td>
<td>1000.3</td>
<td>0.81</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>Assembled powertrain</td>
<td>1</td>
<td>345.1</td>
<td>340.64</td>
<td>0.75</td>
<td>1.29</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>393.9</td>
<td>406.5</td>
<td>0.75</td>
<td>3.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>520.7</td>
<td>511.7</td>
<td>0.80</td>
<td>1.73</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Comparison of natural frequency for powertrain components and its assembly

Due to these results, the FE model of powertrain is well justified for the numerical simulation of its noise and vibration analysis.

4 Simulation of excitation force

4.1 Theory of engine vibration

There are two major forces which excites the powertrain. One is the combustion force, which is caused by combustion pressure and it occurred by firing of air and fuel mixture gas. This combustion force pushes the piston down and is transferred to the crankshaft. And this force is also transferred to cylinder block via main bearing. The other is the inertia force, which depends on the unbalance of the reciprocating mass such as piston or connecting rod.

The combustion force is related to the cylinder pressure. By assuming that the work done by piston is equal to that by crankshaft, Eq.(2) is derived by

\[ p dv = T_p d\theta \] (2)

and

\[ T_p = pA_p \frac{ds}{d\theta} = pA_p \frac{ds}{dt} \frac{dt}{d\theta} = pA_p \frac{\dot{s}}{\Omega} \] (3)

where \( S \) is the displacement of piston movement

\[ S = R(\cos \theta + \frac{\ell}{R} \cos \phi) \] (4)

The velocity of the piston is obtained by the derivative of the displacement and is given by

\[ \frac{\dot{s}}{\Omega} = -R(\sin \theta + 2a_2 \sin 2\theta + 4a_4 \sin 4\theta + \cdots) \] (5)

where \( a_2 = \frac{L}{R} a'_2, \ a_4 = \frac{L}{R} a'_4 \), and
\[ a'_2 = \left[ \frac{1}{4} \left( \frac{R}{L} \right)^2 + \frac{1}{16} \left( \frac{R}{L} \right)^4 + \frac{15}{512} \left( \frac{R}{L} \right)^6 + \cdots \right] \]  
\[ a'_4 = -\left[ \frac{1}{64} \left( \frac{R}{L} \right)^4 + \frac{3}{256} \left( \frac{R}{L} \right)^6 + \cdots \right] \]  

By inserting Eq.(6) and Eq.(7) into Eq.(5), Eq.(8) is obtained as follows:

\[ (\sin \theta + 2a_2 \sin 2\theta + 4a_4 \sin 4\theta + \cdots) = \sin \theta + 2\left[ \frac{1}{4} \left( \frac{R}{L} \right)^2 + \frac{1}{16} \left( \frac{R}{L} \right)^4 + \frac{15}{512} \left( \frac{R}{L} \right)^6 + \cdots \right] \]
\[ + 4\left[ \frac{1}{64} \left( \frac{R}{L} \right)^4 + \frac{3}{256} \left( \frac{R}{L} \right)^6 + \cdots \right] + \cdots \]

From Eq.(3) and Eq.(5) the maximum torque takes place at \( \theta = 90^\circ \) and \( (\sin \theta + 2a_2 \sin 2\theta + 4a_4 \sin 4\theta + \cdots) = 1 \).

The maximum torque is expressed by

\[ T_p = (-)pA_\rho R \]  

where \( p \) is cylinder pressure, \( A_\rho \) is the area of the top part of the piston, and \( R \) is the rotation radius of the crankshaft.

The combustion force is calculated by multiplying the piston area by cylinder pressure. This force is transferred to the mount bracket of the powertrain through crank train system and cylinder block.

The inertia force results from the unbalance of the moving parts of the engine. The major unbalanced inertia force of a single cylinder is given by

\[ F_a = M_\rho \ddot{S} = -M_\rho \Omega^2 R (\cos \theta + \frac{R}{L} \cos 2\theta) \]  

By putting \( Z = M_\rho \Omega^2 R \), the inertia force is given by

\[ F_a = Z (\cos \theta + \frac{R}{L} \cos 2\theta) \]  

The first term in the Eq.(11) is the first order inertia force for the revolution of the crankshaft, and the second term is the second order inertia force. By considering the phase of each cylinder, the total unbalanced force for the in-line 4 cylinder engine, is given by

\[ \sum F_a = Z (\cos \theta_1 + \cos \theta_2 + \cos \theta_3 + \cos \theta_4) \]
\[ + Z \frac{R}{L} (\cos 2\theta_1 + \cos 2\theta_2 + \cos 2\theta_3 + \cos 2\theta_4) \]

By inserting phase differences of each cylinder into Eq.(12), the inertia force is expressed by phase of the first cylinder.

\[ F_a = 4Z \frac{R}{L} \cos 2\theta_1 \]

In order to attenuate the unbalanced force, a balance shaft is applied. In general, two types of balance shafts adopted in the engine: MMC type [9] and Lanchester type [10]. The former reduces the unbalanced bounce force and moment; the latter controls only the bounce force [11]. The Lanchester type is adopted in the test powertrain. Therefore, the moments due to unbalanced forces, which are rolling moment, pitch moment and yawing moment, still exist in the test powertrain.

4.2 Prediction of excitation force

In order to estimate the excitation force, the forced vibration analysis of FE model is required. The FE model of the powertrain created in the previous section is imported to multi-body dynamic analysis software, MSC.ADAMS [12]. The components of FE model are connected with boundary conditions as shown in Fig.7. The vibrations and forces at four mounting points are analyzed through dynamic simulation using MSC.ADAMS.

![Fig.7 Boundary conditions for the assembly of powertrain components](image)

The combustion forces are applied to the top area of the pistons and the inertia forces are calculated in MSC.ADAMS automatically during the simulation.

Fig.8 shows the excitation forces at 2000rpm by the numerical simulation. According to the results, the excitation force at each mount is periodic and the dynamic force at the engine mount is higher than other mounts.

![Fig.8 Excitation forces simulated by CAE technology at four mounting points](image)

5 Validation of the dynamic model

In order to prove validity of excitation force by dynamic analysis, an indirect method is applied because it is difficult to measure the excitation forces at the mounting points directly. Therefore, it is required to compare the measured accelerations through a test with displacements predicted by simulation. Fig.9 shows the displacement and acceleration at the front roll mount bracket during simulation.
And Fig.10 shows the comparison between the simulated displacement and the measured displacement at 2000rpm in the frequency domain. According to these results, the simulated displacements are very close to the measured displacements. Some differences are due to engine friction force and modeling error of bearings. It is difficult to estimate the displacement exactly because the powertrain structure is complex and there are lots of components as well as connections in powertrain. These errors are natural and many trials are required to match the simulated results to the measured results exactly.

**Fig. 10** Comparison between the measured displacement and the simulated displacement at powertrain mounting points: (a) E/G mount (b) F/R mount (c) R/R mount (d) T/M mount

6 Conclusion

In this paper, the displacements, accelerations and forces at powertrain mounting points are simulated by CAE technology based on the FE model. The simulated excitation force will be used as the input force of the vehicle body for prediction of interior noise caused by structure-borne path by hybrid TPA. The FE model of powertrain components are validated experimentally. The dynamic analysis is performed to predict the vibration at the powertrain mounting points using this FE model. The predicted displacement is compared with measured displacement at four mount points of powertrain. The results are very close with a minor error. According to these results, it is carefully inferred that the simulated force based on the CAE technology can be also used for the prediction of the interior noise caused by structure-borne path of the powertrain vibration based on the hybrid TPA with accuracy.

**Acknowledgments**

This work was supported by Automobile Component Base Technology Project No.10023237 in Korea.

**References**


