We investigate, both experimentally and theoretically, the spectral properties of the backscattering operator in random media. The experimental set up consists in an array of 64 programmable ultrasonic transducers placed in front of a random scattering medium. The impulse responses between each couple of transducers are measured and form the interelement matrix. The evolution of the singular values with time and frequency is computed by means of a short-time Fourier analysis. Our aim is to compare the mean distribution of the singular values with the predictions of Random Matrix Theory (RMT). Two kinds of random media have been investigated: a highly scattering medium in which multiple scattering predominates and a weakly scattering medium. In both cases, residual correlations that may exist between matrix elements are shown to be a key parameter. The mean distribution of singular values exhibits, surprisingly, very different behaviors in the single and multiple scattering regimes. The agreement between experimental results and RMT predictions is found to be very good in both cases. The results are applied to the detection of a target embedded in a random scattering medium with the DORT method.