There is an approach, named the vector-phase method, based on enhanced amount of the acoustic field information extracted from each point of the field at the present samples volume. It is realized by setting both receivers of sound pressure and gradient pressure ones simultaneously in the limited number of medium points. The application of these methods for the solving of practical acoustics problems is discussed. These problems include the following: the detection of the characteristic sizes of small-scale inhomogeneity in ocean; the detection of signals from determinate sources at signal/noise ratio far less than one; separation of noise contributions from the disturbed surface ocean and removed sources, based on ratio measurement of sound pressure and vertical component of oscillatory velocity; classification of the acoustic sources, based on the analysis of difference-phase relation between pressure and oscillatory velocity; definition of directional characteristic of source by simultaneous measurements of pressure and the normal component of gradient pressure on closed surface. The vector-phase method is effective for studying of reflecting properties of multilayered bottom at extremely low frequencies. This is due to complex data registering, both amplitude and difference-phase ratio of pressure and components of oscillatory velocity.

1 Introduction

This work is considered with the methods of decision of certain important problems in the low-frequency hydroacoustics, based on enhanced amount of the acoustic field information at the constant sample volume due to simultaneous registration of sound pressure and amplitude and phase of oscillatory velocity of medium particles in a wave in the same points of area. In practice, it is possible more precisely to measure the sound pressure gradient, to which the oscillatory velocity of medium particles in a wave is proportional within some complex factor, or oscillatory acceleration. These measured characteristics of acoustics field differ in principle from pressure field, because they are vector characteristics. All methods based on such approach [1,2], are named the vector-phase methods now. The design of sound receiver should consist of some independent channels of information registration since for definition of vector in space it is necessary to have its three characteristics. Thus, this approach of obtaining of the additional information about acoustic field can be realized by means of disposition in the limited number of medium points both receivers of sound pressure (zero order receivers) and vector receivers (first order receivers), which allow to define components of pressure gradient, oscillatory velocity, acceleration of medium particles in an acoustic wave. Other peculiarity of the local combined reception system is an opportunity of direct measurement of acoustic power flux, i.e. determination of that part which is caused by field anisotropy or presence of the determinate sources in the spatial region. Besides, simultaneous registration of several components of a field without amplitude-phase distortions allows analyzing of character of particles movement in a wave (the polarizing analysis) in order to provide classification of sources. It is very important if vector receivers are placed on bottom or earth crust (like geophone).

The experimental and theoretical researches made for a long time at acoustics department of Physics Faculty of Moscow Lomonosov State University have allowed formulating a number of approaches to the decision of some important problems of acoustics. The application of these methods for the solving of practical acoustics tasks is being discussed in this paper.

2 The Basics of Vector-Phase Methods

2.1 Classification of acoustic sources

Classification of sound sources, definition of sources strength and directional characteristic, measured by characteristics of a field near source surface have a big practical value. Directional characteristic of source is the characteristic of a distant field. Therefore traditional measurement is necessary to be carried out at the distances essentially exceeding the sizes of the source, and to observe conditions of a free field. Increase of the sources sizes and decrease of working frequencies makes distances become rather greater. Because of medium heterogeneity at such distances effects caused by refraction and sound scattering can arise. Besides, influence of the uncontrollable conditions, surrounding noise, surface and bottom in particular, appear. In fact, necessity of distance increase between source and receiver system has made impossible using of the anechoic chamber and hydro
2.2 Calculation of directional characteristic by measurements in a near field

For calculation directional characteristic of source by measurements in a near field it is convenient to use Helmholtz’s integrated theorem. If there is a system of sources of sound localized inside of some area, limited closed surface \( S \), pressure in every point of \( M \) located outside of this surface is expressed by the following integrated formula:

\[
P(M) = \sum_{S} \left\{ P(N) \frac{\partial G(N)}{\partial n} - \frac{\partial P(N)}{\partial n} G(N) \right\} dS,
\]

\( N \) are the points at closed surface \( S \); \( P(N) \) are complex values pressure and a normal derivative of pressure in point \( N \); \( G(N) \) is Green’s function. This method allows defining directional characteristic of source in any plane due to one sample of field parameters \( P \) and at control surface without reorientation of source. This method is characterized by high accuracy of measurements and is antijamming because presence of extraneous sound sources and reflections from borders is not essential in its realization. The technique has been checked up in anechoic chamber experimentally. For measurement of pressure and its derivative microphone and gradient pressure receiver were used, they have been fixed in special designed frame established at grid of anechoic chamber. Experimental data are in good conformity with results of mathematical modeling.

2.3 Definition of strength \( Q_0 \) of determinate source

Definition of strength \( Q_0 \) of the determinate source in the majority of practical cases is solved measurement of sound pressure in free space conditions at accommodation of the reception system in a distant field of a source, where the phase difference between pressure \( P \) and oscillatory velocity \( V \) is equal to zero, or in a near field of a source by using of analytical relation between pressure and strength. In a low-frequency range as a rule it is not possible to create conditions of free space, therefore for definition \( Q_0 \) it is necessary to take into account addition reaction of borders or to do measurements in open deep space in noise field created both disturb surface, and removed determinate sources. For example, in conditions of a plane waveguide depth \( H \) it is possible to receive rather simply theoretical dependences, \( P = Q_0 \Psi_P(r, d_z, z, H) \), \( V_r = Q_0 \Psi_y(r, d_z, z, H) \), \( W = Q_0 \Psi_w(r, d_z, z, H) \), \( d_z \) and \( z \) depths of submergence of receiver and source, \( V_r \) horizontal component \( V \). Calculation of exact real value of multipliers \( \Psi \) represents rather difficult operation, since small mistakes of assignment of reception point coordinates and also fluctuations of other parameters, which are included in these multipliers, lead to significant mistakes of calculations. The first variant the decision of this task is averaging of the data of components measurements \( P \) and \( V_r \) alone vertical line. In this case expressions for \( \Psi \) become essentially simpler.
The second way is measurement of acoustic power flux, which, in case of absence of losses in the medium allows to experimental work at any distances from source, to get rid of reactive density of the sound energy generated by borders, which in the given conditions should be considered as disturbance. Integral theorem Ostrogradski-Gauss is the basis of the method. With reference to the determinate sources like monopole it can be formulated as follows: «the full acoustic power flux through any closed surface \( S \) is proportional to the sum of strengths of the sources situated inside this surface». It is rather essential, that the method is antijammed in relation to the sources of a noise which are outside of surface \( S \), as for them this flux appears equal to zero. For example, for pulsing sphere radiating power we have:

\[
W_R = \frac{1}{T} \int_0^T P(t) t(t) dt = \frac{1}{2} P_0 Q_0 \cos \varphi_{PV}
\]

\[
Q_0 = \sqrt{-\frac{2W_R}{\rho \cdot \text{Im} \Phi_0}}
\]

Im \( \Phi_0 \) for oscillatory velocity scalar potential of a source with unit strength on a source surface one can find by comparison method at presence calibration source, or by calculation. Value \( W_R \) can be found by measuring of acoustic power flux through any closed surface surrounding the source. The value determined thus does not depend on character of borders and presence of sources outside of a surface. For plane layer of water with parallel borders as a surface of integration \( S \) it is convenient to use cylinder with radius \( a \) and with an axis passing through source, and top and bottom bases coincide with the top and bottom borders of a layer. In this case the full acoustic power flux follows through a lateral surface of the cylinder and through the bottom basis.

### 2.4 Research of reflecting properties of a bottom

Let’s consider methods of research of reflecting properties of a bottom at extremely low frequencies by simultaneously registering both amplitude and difference-phase ratio of pressure and projections of oscillatory velocity [3].

**Impedance method** is based on direct measurement of relation between pressure and vertical component of oscillatory velocity in one point on the water/ bottom boundary. It is known, that for a layered bottom the entrance impedance of a bottom is defined from the following recurrent formula:

\[
Z_{in} = Z_n\left(\rho_n c_n^{-1} - j \cdot Z_n \cdot \tan \varphi_n \right) \left(Z_n - jZ_{in}^{-1} \cdot \tan \varphi_n \right);
\]

\[
j = \sqrt{-1}; \quad Z_n = \rho_n c_n / \cos \vartheta_n; \quad \varphi_n = \beta_n \cos \vartheta_n / c_n;
\]

\( \rho_n, c_n \) density of bottom layer with number \( n \) and sound speed in it, \( h_n \) thickness of this layer, \( Z_{in}^{-1} \) entrance impedance of \( n \) layers below, \( c \) is sound speed in water. The angle \( \vartheta_n \) is defined from the formula:

\[
c \cdot \sin \vartheta = c_n \sin \vartheta_n = ... c_1 \cdot \sin \vartheta_1.
\]

The impedance and reflection coefficient \( V(\vartheta_f) \) depend on angle incidence to the bottom and frequency are connected by a relation:

\[
V(\vartheta, f) = \left[ Z_n(\vartheta, f) - \frac{Z_0}{\cos \vartheta_n} \right] \left[ Z_n(\vartheta, f) + \frac{Z_0}{\cos \vartheta_n} \right].
\]

Entrance impedance \( Z_0 \) can be determined by measurements of relation between pressure and vertical component of oscillatory velocity in one point on the water/ bottom boundary.

**Method** is based on measurement phase difference between pressure and horizontal or vertical component of particle oscillatory velocity. Principally it is possible to define parameters of a bottom by such method in any conditions for which one can find the relation between horizontal \( V_x \) or vertical \( V_z \) components of oscillatory velocity and sound pressure \( P \). However it is most expedient such measurements carry out at plane bottom, where relation between \( V \) and \( P \) are the most simple:

\[
Z_{in} = \left( \frac{P}{V_z} \right)_{\vartheta = \vartheta_n}.
\]

When measuring phase difference between pressure and horizontal component \( V_x \), the result of measurement depends from insignificant area of a surface. Obviously, that for realization of this method it is necessary to allocate a separate normal mode that is the most simple by using the frequencies close to critical.

**Method** in which *additive combination* of sound pressure and vertical component of oscillatory velocity are used. The essence of this method consists in formation of minima of reception system directional characteristic in the direction at the signals extending from a surface to a bottom (numerator) and, on the contrary, reflected from bottom (denominator):

\[
V(\vartheta, f) = (P \cos \vartheta - V_x) / (P \cos \vartheta + V_x).
\]

Absence of necessity of reception system moving along vertical line during measurements essentially reduces the time of measurements. Besides, there is an opportunity of studying thin frequency structures of reflection coefficient by using of a broadband source with the narrow-band analysis of signal sample received for all spectrum frequencies simultaneously. Presence of obviously expressed minima at frequency dependence of the module of reflection coefficient \( |V(\vartheta_f)| \) testifies layered structure of a bottom.

**Method** of stationary waves (interference method), consisting in measurement of interference structures of a field near bottom is the most exact method of
measurement of reflection coefficient. Using only a field of pressure the method does not demand special calibration of source and receiver and can be used in enough wide frequency range as only relative values of levels registered in maxima and minima of interference picture are used:

\[ |V| = \frac{P_{\text{max}} - P_{\text{min}}}{P_{\text{max}} + P_{\text{min}}}. \]

On extremely low frequencies there is an error caused by effects of a near field. In this case, using both deep sections of a pressure field and of vertical component field of oscillatory velocity \( V_z \) is useful. Calculation of the module of reflection coefficient is made in accordance with the formulas:

\[ |V| = \frac{P_{\text{max}} - V_{z\text{ min}}}{P_{\text{max}} + V_{z\text{ min}}}, \quad |V| = \frac{V_{z\text{ max}} - P_{\text{min}}}{V_{z\text{ max}} + P_{\text{min}}}. \]

### 2.5 Determination of the characteristic sizes of small-scale inhomogeneity in ocean

The method is based on radiation of tonal acoustics signal with frequency \( f \), reception at such distance \( r \), that \( kr >> 2a/\lambda \), where \( a \) is characteristic size of inhomogeneity and on simultaneous measurement of amplitude of sound pressure, two orthogonal components of oscillatory velocity on horizontal plane, phase difference between them and sound pressure. Further, bearing angle \( \phi \) of sound signal source and a dispersion of bearing for the same time are determined, and the characteristic size of small-scale inhomogeneity is calculated by the formula:

\[ a = 2\sqrt{\langle \mu^2 \rangle} \cdot \frac{r}{\sigma_\phi}. \]

\( <\mu^2> \) is dispersion of relative fluctuation of sound speed, \( \sigma_\phi^2 \) is dispersion of bearing angle \( \phi \) in the direction of sound signal source.

### 2.6 Separation of noise contributions from the disturbed ocean surface and removed sources

For this purpose in the same point at the depth \( d_s \), such as \( kd_s > 3 \) in all investigated frequency range, sound pressure and three orthogonal components of oscillatory velocity are measured simultaneously. For frequencies where condition

\[ P_i^2/(V_x^2 + V_y^2 + V_z^2) \approx (\rho \sigma)^2 \]

is satisfied, i.e. there are no hydrodynamic noise, and in range \( r < 10 \ldots 15 \) \( d_s \), where there are no determinate sources, the following relation is carried out:

\[ \frac{P^2}{P^2_n} \approx \frac{P^2}{(\rho \sigma V_z)^2} \cdot \frac{n}{n+1}, \]

\( P^2_n \) is the contribution of removed sources noise into the acoustic field \( P_n^2 \), which was generated by the disturbed surface, \( n \) is the characteristic of direction of surface noise. In the majority of practical cases it is possible to suppose that \( n \approx 1 \).

### 2.7 Allocation of weak level signals from determinate sources on background of ocean noises

The method is founded on registrations of mutually orthogonal in horizontal plane of acoustic power flux components in the given frequency range with beforehand narrow-band spectrum analysis:

\[ W_{\text{nl}}(f) = \frac{1}{\tau} \int_0^\tau P(t, f) \cdot V(t, f) \, dt \quad (i=x, y) \]

Then statistical provided histogram bearing angle \( \phi - \) intensity \( I(\phi) = \sqrt{\frac{W_x^2 + W_y^2}{\langle \mu^2 \rangle}} \) is calculated. Since for dynamic noises of ocean the value of \( P \) and \( V \) are weakly correlated, it is possible to locate and separate signals if the relation signal/noise on field of pressure from \(-12 \ldots -15 \) dB for coast shallow regions to \(-25 \) dB for areas removed from navigable line of deep ocean.

### 3 Summary

The given above material witnesses that developed in Moscow Lomonosov State university vector-phase methods at present are important and contribute significantly to the improvement of hydroacoustic methods for decision of applied problems of acoustics.

### References

