Uncertainties in building acoustics

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The uncertainties associated with the airborne sound insulation are investigated. Starting point is an analysis of the different influence factors, which finally results in the identification of a total of 16 uncertainty contributions. The order of magnitude of some uncertainty contributions is estimated on the basis of the data available. In addition to intercomparison results available from literature, the estimate also covers internal PTB data.

The individual uncertainty contributions are assigned to the precision measures so far used in building acoustics, i.e. repeatability and reproducibility according to ISO 140-2, which allows the existing intercomparison results to be quantitatively compared with the values from ISO 140-2. It turns out that the values stated in ISO 140-2 are usually lower than the actually occurring uncertainties.

In conclusion, the consequences resulting from the uncertainties for the safety margin and for re-measurements are shown. It is furthermore investigated in detail how many different specimens of a building element have to be measured in how many different test facilities to achieve that the uncertainty assigned to the mean value from these measurements does not exceed specified upper limits. In addition, the propagation of the uncertainties from the values of the building element catalogue up to the prognosis value is calculated on the basis of an example. It results that the uncertainty of the predicted value is still lower than, or maximally equal to, the individual uncertainties of the building elements involved.

1 Introduction

An assessment of uncertainties which is comprehensible and close to reality is indispensable for many questions in building acoustics. Whether a requirement is met, a laboratory delivers correct results or the acoustic properties of a product are better than the same properties of some other product can be decided only by adequately assessing the uncertainties associated with the quantities under consideration. Due to the actuality and large impact of these questions, the Deutsches Institut für Bautechnik (DIBt), the regulatory authority for all building matters in Germany, allocated a project to the Physikalisch-Technische Bundesanstalt (PTB) whose main results will be outlined in this contribution.

2 Test on Gaussian distribution

Before a detailed model for the calculation of uncertainties is developed, it seems necessary to investigate whether all considerations should be based on a dB-scale or on physical quantities. The results of a round robin using a heavy lime-brick wall [1] are used to answer this question. For this purpose, the primary data base - consisting of results from 12 laboratories - has been enlarged by the results of the same wall from 8 further laboratories. The distribution of third-octave values and of the weighted sound reduction indices of the 20 laboratories was tested against the hypothesis of a Gaussian distribution by the Kolmogorow-Smirnow-test. Besides the sound reduction index \( y \), the ratio between inserted and transmitted sound power \( W_1/W_2 \) and the reciprocal of this quantity were included in the test. The sound reduction index is named \( y \) throughout this contribution to avoid confusion with the reproducibility \( R \).

Figure 1: Empirical distribution of the sound reduction index \( y \) and of the power ratios \( W_1/W_2 \) and \( W_2/W_1 \) in comparison with the Gaussian distribution function, third-octave values at \( f = 50 \text{ Hz} \)
applies to all frequencies as well as to the weighted sound reduction index. A concentration on level quantities seems therefore appropriate.

3 Uncertainty contributions to the sound reduction

A two-channel simultaneous measurement of the sound pressure levels in the sending and receiving rooms is considered for the determination of relevant uncertainty contributions. The sound reduction index $y$ is then

$$y = L_{p,1} - L_{p,2} - K_{\text{abs}} - K_{\eta} - K_{\text{SLM1}} - K_{\text{SLM2}} - K_{\text{av1}} - K_{\text{av2}} - K_{\text{W1}} - K_{\text{W2}} - K_{\text{pos}} - K_{\text{met}}$$

where the difference between the mean sound pressure levels $L_{p,1}$ and $L_{p,2}$ is corrected for

- the absorption in the receiving room $K_{\text{abs}}$
- the background noise in the receiving room $K_{\eta}$
- influences of the measurement equipment used to measure the sound pressure levels in the receiving and in the sending room $K_{\text{SLM1/2}}$
- imperfect spatial and temporal averaging $K_{\text{av1/2}}$
- deviations from the ideal sound field $K_{\text{W1/2}}$
- influences of the source position(s) $K_{\text{pos}}$
- the meteorological conditions $K_{\text{met}}$
- the aspect ratio of the test specimen $K_{\text{ar}}$
- the size of the test specimen $K_{\text{size}}$
- the energy transmission to the test suite $K_{\eta}$
- the boundary conditions (rigid, elastic, ...) $K_{\text{boun}}$
- the maximum sound reduction of the test suite $K_{\text{max}}$ and
- the limited reproducibility of the building element $K_{\text{repro}}$

The uncertainty of the sound reduction index is then, according to [2],

$$u(y) = \sqrt{\sum_{i=1}^{16} u_i^2 \left( K_i \right)}$$

whereby correlation between input quantities has been neglected. A detailed analysis of the uncertainty contributions revealed that at the present state of knowledge, only some of them can be quantified. It therefore makes sense to relate the model function (1) to the more global precision measures having been used in building acoustics so far.

4 Classification of the uncertainty components

Single uncertainty components can be divided into three main groups. The first group comprises the contributions occurring when measurements are repeated on the same object and in the same test suite. The sum of all these uncertainty contributions can be regarded to be equivalent to the standard deviation of repeatability $\sigma_r$ used in ISO 140-2 [3]

$$\sigma_r = \left[ u^2 (K_b) + u^2 (K_{av1}) + u^2 (K_{av2}) + u^2 (K_{\text{pos}}) + u^2 (K_{\text{met}}) \right]^{1/2}$$

A value for this quantity has been determined by analyzing the data of the comparison measurements, which are held regularly at PTB and MPA Dortmund. During these measurements, 10 (PTB) respectively 100 (MPA) measurement teams determine the airborne sound insulation of the same object in the same test suite. From all these measurements, an upper limit for the standard deviation has been determined. The result is significantly larger than the values from ISO 140-2 or from different round robin tests (Figure 2).

This discrepancy can be explained by two different effects. The first one is that the newly determined values clearly contain the uncertainty contribution from the measuring instrument, whereas this contribution is not included in $\sigma_r$ as defined in ISO 140-2. Nevertheless, in order to explain the difference quantitatively, this single uncertainty contribution should amount to about one dB at frequencies above 0.5 kHz and to even larger values at smaller frequencies. Since this seems to be unlikely it is assumed that the microphone and source positions have not been varied in such a way that all the regions tolerated by the standard have been covered in the round robin tests. The assumption that “good” laboratories have a small standard deviation of...
repeatability often led to only small variations between the repeated measurements.

The second group of uncertainties comprises systematic deviations between the mean values from different laboratories. It is called inter-laboratory standard deviation $\sigma_I$

$$\sigma_I = \left[ u^2 (K_{SLM1}) + u^2 (K_{SLM2}) + u^2 (K_{W1}) + u^2 (K_{W2}) + u^2 (K_{meas}) + u^2 (K_{ar}) + u^2 (K_{size}) + u^2 (K_{f}) + u^2 (K_{boun}) + u^2 (K_{max}) \right]^{1/2} \tag{4}$$

The standard deviation of reproducibility $\sigma_R$ is now

$$\sigma_R = \sqrt{\sigma_r^2 + \sigma_L^2} \tag{5}$$

The $\sigma_r$-values from ISO 140-2 are nearly equal to the mean value from different round robin tests (Figure 3).

![Figure 3: standard deviation of reproducibility from ISO 140-2 and various round robin tests](image)

The third group of uncertainties characterizes influences which are due to the limited reproducibility of the building element under test. These effects have also been investigated within the scope of the project. It turned out that the standard deviation $\sigma_{repro}$ is about 1 dB for building elements which can be reproduced easily such as the lime-brick wall from the round robin or glazings. A typical value for building elements difficult to be reproduced, as for example double-leaf gypsum board walls, is about 3 dB. But it must be pointed out that all data available on this subject stem from round robin tests. This means that test objects are specially designed to minimise the effects of limited building element reproducibility. The results must therefore be considered to be at the lower end of possible values.

5 Consequences following from the uncertainties

5.1 Determination of product-related mean values

A first interesting question is now how many measurements are necessary to state a sound reduction index of a building element with a defined uncertainty. The uncertainty of the mean value from $n_1$ repeated measurements in the same test suite obtained with the same test specimen is

$$u_1(\bar{Y}) = \sqrt{\frac{\sigma^2_Y - \sigma_r^2}{n_1} + \sigma_{repro}^2} \tag{6}$$

If, however, $n_2$ repeated measurements are carried out in the same test suite at $n_2$ different test specimen, the uncertainty is

$$u_2(\bar{Y}) = \sqrt{\frac{\sigma^2_Y - \sigma_r^2}{n_2} + \frac{\sigma_{repro}^2}{n_2}} \tag{7}$$

For $n_3$ measurements in $n_3$ different test suites with $n_3$ individual test specimens in each test suite, the uncertainty finally is

$$u_3(\bar{Y}) = \frac{1}{\sqrt{n_3}} \sqrt{\sigma_r^2 + \sigma_{repro}^2} \tag{8}$$

The required number of measurements can now be calculated for the different cases by eqs. (6) - (8). The calculation is carried out for the weighted sound reduction index where it is assumed that the respective standard deviation of reproducibility is 2 dB and of repeatability 1 dB. These values are significantly larger than the values from ISO 140-2 ($\sigma_R = 0.4 \ldots 1.1$ dB, $\sigma_r = 0.4$ dB), but they seem to be justified for example by the results from the heavy lime-brick wall, where $\sigma_R$ was 2.4 dB. Effects of limited reproducibility of the building element are estimated to be 1 or 3 dB.

Calculation results are shown in Table 1. An uncertainty of 1.0 dB is only reached when different test suites are used whereby 5 measurements are necessary for $\sigma_{repro} = 1$ dB and 13 for $\sigma_{repro} = 3$ dB. In general, repeated measurements at the same test specimen in the same test suite do not reduce the uncertainty of the mean value significantly since the standard deviation of repeatability delivers only a minor contribution to the combined uncertainty.
A further possibility of determining the product-related standard deviation is to repeat the measurements \( N \) times for each specimen. For a constant number of repetitions \( N \), the standard deviation of the mean values from these measurements turns out to be

\[
\sigma = \frac{\sqrt{\sigma_y^2} + \sigma_{\text{repro}}^2}{N}.
\]  

If reasonable values are assumed for \( \sigma_{\text{repro}} \) and \( \sigma_y \), the required number \( N \) can be calculated by eq. \( (11) \) in analogy to Table 1.

Finally, the question of the required number \( M \) of test specimens shall be addressed. The statistical procedure used is the estimation of a standard deviation of a basic population from a sample. The confidence interval of such an estimate can be calculated using the \( \chi^2 \)-distribution, and to obtain justifiable results, at least 5 specimens must be used.

### 5.3 Propagation of uncertainties to the predicted value

Starting points for the prediction are now the mean values of the building elements which contribute to the sound propagation in the given situation and the corresponding uncertainties.

\[
\bar{y} \pm u(\bar{y}).
\]

These values characterize the mean performance in a mean laboratory situation if they have been determined according to 5.1. But the prediction is related to a single specimen, and therefore the values must be converted. For the sound reduction, the mean value is the best estimate

\[
y = \bar{y}
\]

whereas for the uncertainty, the reproducibility of the building element has to be included

\[
u(y) = \sqrt{u^2(\bar{y}) + \sigma_{\text{repro}}^2}.
\]

These values are now characteristic of a single specimen in a mean laboratory situation.

The next step is the transition to a specific building situation. This is achieved by an appropriate correction

\[
y_{\text{bul}} = y - K_{\text{bul}}.
\]

The value of this correction cannot be stated at the present state of knowledge. There are some reasons why the sound reduction index tends to be slightly larger in laboratories than in building situations. One reason is that laboratories tending to higher values are favored by industry; another one could be that asymmetric configurations as used in laboratories usually lead to higher sound insulations than the symmetric configurations which are often met in practice. The uncertainty of this value is now

#### Table 1: Number of measurements required for given uncertainties \( u \)

<table>
<thead>
<tr>
<th>( u ) target</th>
<th>( \sigma_{\text{repro}} = 1 \text{ dB} )</th>
<th>( \sigma_{\text{repro}} = 3 \text{ dB} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_1 )</td>
<td>( n_2 )</td>
<td>( n_3 )</td>
</tr>
<tr>
<td>1.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3.0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
\[ u(y_{\text{Bau}}) = \sqrt{u^2(y) + u^2(K_{\text{Bau}})} \]  \hspace{1cm} (16)

A lower limit for the uncertainty of the correction is the inter-laboratory standard deviation, since all laboratory situations are a subset of all building situations

\[ u(K_{\text{Bau}}) \geq \sigma_L. \]  \hspace{1cm} (17)

The prediction result is now determined by combining all the \( n \) values contributing to a sound transmission, which is expressed by a function \( f \) here

\[ y_{\text{pred}} = f(y_{1,\text{Bau}}, y_{2,\text{Bau}}, \ldots, y_{n,\text{Bau}}). \]  \hspace{1cm} (18)

The uncertainty of this value can then be determined by applying the principles of [2] to the prediction equation (18). This is demonstrated here for the very simple example that the in-situ sound reduction is calculated for a structure consisting of \( n \) elements, e.g. a wall with a door and some windows. The sound reduction index is then

\[ y_{\text{pred}} = 10 \log \left[ \frac{\sum_{i=1}^{n} S_i}{\sum_{i=1}^{n} S_i 10^{-y_{i,\text{Bau}}}/(10 \text{dB})} \right] \text{ dB}. \]  \hspace{1cm} (19)

with the size and in-situ sound reduction of the \( i \)-th element \( S_i \) and \( y_{i,\text{Bau}} \), respectively. The uncertainty of the predicted value is then, according to [2],

\[ u(y_{\text{pred}}) = \frac{1}{\sqrt{n}} \left[ \sum_{i=1}^{n} c_{S,i} u(S_i) + \sum_{i=1}^{n} c_{y,i} u(y_i) \right]. \]  \hspace{1cm} (20)

Correlation between input quantities is neglected here. The so-called sensitivity coefficients turn out to be

\[ c_{S,i} = \frac{\partial y_{\text{pred}}}{\partial S_i} = \frac{10^{y_{i,\text{Bau}}}/(10 \text{dB})}{\ln 10} \left[ \frac{1}{\sum_{i=1}^{n} S_i} \cdot \frac{S_i}{\sum_{i=1}^{n} S_i 10^{-y_{i,\text{Bau}}}/(10 \text{dB})} \right] \]  \hspace{1cm} (21)

and

\[ c_{y,i} = \frac{\partial y_{\text{pred}}}{\partial y_i} = \frac{S_i 10^{-y_{i,\text{Bau}}}/(10 \text{dB})}{\sum_{i=1}^{n} S_i 10^{-y_{i,\text{Bau}}}/(10 \text{dB})}. \]  \hspace{1cm} (22)

Table 2: Uncertainty budget for the prediction

<table>
<thead>
<tr>
<th>quantity</th>
<th>estimate</th>
<th>distribution</th>
<th>( u_i )</th>
<th>( c_i )</th>
<th>( u_i \cdot c_i )</th>
<th>dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_{\text{W,buil}} )</td>
<td>53 dB</td>
<td>Gauss</td>
<td>3 dB</td>
<td>0.386</td>
<td>1.16</td>
<td>dB</td>
</tr>
<tr>
<td>( y_{\text{D,buil}} )</td>
<td>44 dB</td>
<td>Gauss</td>
<td>3 dB</td>
<td>0.614</td>
<td>1.84</td>
<td>dB</td>
</tr>
<tr>
<td>( S_{\text{W}} )</td>
<td>10 m²</td>
<td>Gauss</td>
<td>0.5 m²</td>
<td>0.362 dB/m²</td>
<td>0.18</td>
<td>dB</td>
</tr>
<tr>
<td>( S_{\text{D}} )</td>
<td>2 m²</td>
<td>Gauss</td>
<td>0.1 m²</td>
<td>0.362 dB/m²</td>
<td>0.04</td>
<td>dB</td>
</tr>
<tr>
<td>( y_{\text{pred}} )</td>
<td>50 dB</td>
<td></td>
<td>2.2 dB</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Let us consider the example that the sound reduction of a wall with a door inside is to be predicted. The whole calculation procedure and the values used can be found in Figure 4. The uncertainty budget for the last step, the prediction according to eqs. (19) to (22), is given in Table 2. The combined uncertainty of the predicted value turns out to be 2.2 dB. As expected, the uncertainty of the size plays only a minor role for the combined uncertainty of the predicted value. Therefore these contributions can be neglected under normal circumstances.

Figure 4: propagation of uncertainties for the example

It is furthermore interesting that the uncertainty of the predicted value is smaller than the uncertainties of the input values. This is due to the fact that the predicted value is a weighted mean value of the input values. In
general, two extreme cases can be observed. The first one is that only one building element dominates the sound insulation. In this case, the combined uncertainty of the predicted value will be equal to the uncertainty of this dominating element. The second case is that all $n$ building elements contribute equally to the predicted sound insulation and have the same uncertainty. The combined uncertainty of the predicted sound insulation will then be

$$u(y_{\text{pred}}) = \frac{u(y_{\text{pred}})}{\sqrt{n}}.$$  
(23)

All situations in practice will be between these two extreme situations.

5.4 Safety margin

The safety margin is necessary to take the uncertainty into consideration by subtracting a multitude of the uncertainty in such a way that values met in practice will be higher than the predicted value with a certain confidence level.

$$y = y_{\text{pred}} - k u(y_{\text{pred}}).$$  
(24)

The expansion factor $k$ is 1 for a confidence level of 84% and 2 for a confidence level of 97.5% (one-sided confidence interval).

5.5 Uncertainties for in-situ re-measurements

Another interesting aspect is the inclusion of uncertainties in in-situ measurements. Such measurements are carried out under repeatability conditions if the influence of the measuring instrument is neglected. The standard deviation for these measurements can be described for example by the values derived from the comparison measurements, provided that the situation is free from special difficulties as, for example very small or partially open rooms. The measurement result is called $y_{\text{meas}}$ here and its expanded uncertainty $U = k u$ has the value

$$U = 2 \sigma_r.$$  
(25)

for a confidence level of 95% (expansion factor $k = 2$). All uncertainty contributions caused by the size of the building element, the aspect ratio, the loss factor, the boundary conditions, the sound field properties etc. should appropriately be taken into account by the safety margin and are invariant for a given situation.

How the expanded uncertainty must be applied depends on the exact question to be answered by the measurement. A possible statement is: the in-situ sound reduction is smaller than the given value $y$. To verify this statement, it must be required that

$$y_{\text{meas}} - U < y.$$  
(26)

Another statement is: the sound reduction is higher than the given value $y$. It is verified by

$$y_{\text{meas}} + U > y.$$  
(27)

If the declared value is within the confidence interval

$$y_{\text{meas}} - U < y < y_{\text{meas}} + U,$$  
(28)

neither the first nor the second statement can be verified.

6 Summary and future work

The investigation led to a deeper understanding of the uncertainties and their implications in building acoustics. Some important questions are still to be solved though. One outcome is for example that many quantities in building acoustics lack generally-accepted precise definitions. This makes discussions on uncertainties really difficult. It is furthermore necessary to investigate the uncertainties of single number values in more detail because they are used for declaration and prediction purposes. Especially the question of correlation between neighbouring third-octave values and consequences for the uncertainties of single number values should be investigated.

Acknowledgements

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