A Modal Approach to Lightweight Partitions with Internal Resonators

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Theoretical models of sound transmission through partitions are well known as far as homogeneous materials are concerned. However, in reality only a small fraction of materials are literally homogeneous. In recent years, considerable interest has been taken in theoretical models of inhomogeneous structures. First results have revealed the opportunity to modify the sound transmission loss at selected frequencies due to inhomogeneous material properties. Using the concepts of forced inhomogeneities, increasing efforts have been made into the development of new materials or structures with higher sound transmission loss at certain frequencies. This paper presents the implementation of local resonance systems into lightweight partitions in order to improve the sound insulation of lightweight structures at lower frequencies. The benefits of double-panel partitions are combined with the frequency dependent effect of distributed vibration neutralizers. The main focus is placed on the location and number of vibration neutralizers. Based on the modal expansion method a beam model has been derived taking the modal behaviour of both panels into account. The radiated power has been calculated using modal radiation efficiencies. The effects of the vibration neutralizers have been quantified in terms of the resulting sound transmission loss. Results are presented of parameter studies that demonstrate the behaviour of lightweight structures dependent on the properties, the distribution and number of the internal vibration neutralizers.

1 Introduction

It is well known that lightweight structures tend to vibrate more easily and therefore radiate more sound. In many fields of mechanical engineering design, absorbing or damping materials have been applied to minimise noise emission at the expense of significant weight penalties. Furthermore, these noise control solutions have only a small effect on low-frequency noise problems.

The traditional simplified approach to estimate the sound transmission through single- or double-panel partitions is the "mass law", in which the mass per unit area is considered as the principal variable in sound transmission loss design. In recent years, research has focused on the development of new physical principles for noise control. For this purpose, considerable interest was taken in the fundamental limitations of Cremer’s theory that demonstrated the existence of the critical frequency [1]. This theory is only valid for a thin homogeneous, elastically isotropic plate with equal fluids on both sides. Maysenhölder [2, 3] has modelled a structure with embedded local resonant mass-spring systems as an idealised thin plate with inhomogeneities periodically arranged. Initial results presented have revealed new significant phenomena in the sound transmission characteristic caused by periodic inhomogeneities. In order to use inhomogeneities to improve the sound transmission loss at low frequencies the idea of forced resonant inhomogeneities came about. By that means the benefits of double-panel partitions were combined with the frequency-dependent effect of distributed vibration neutralizers.

Since the vibration neutralizer was introduced by Ormondroyd and Den Hartog [4], it has been widely used for structural vibration and noise control. Whilst some applications involve a vibration neutralizer to control vibrations of a structure at the point where it is attached, in more recent applications the vibration neutralizer is designed to control sound radiation from vibrating structures [5]. For this reason, vibration neutralizers have been implemented for the reduction of sound transmission through partitions. However, published investigations have been carried out only on single panel structures [6]. In this paper, the implementation of distributed vibration neutralizers into the cavity of finite double-partitions is presented. A beam-model has been derived under the condition that location and number of vibration neutralizers are taken into account. The sound transmission loss has been calculated by the ratio of incident and radiated energy. Since the beam-model has been derived using the modal expansion method, the radiated sound was determined by the modal radiation efficiencies of a beam, published by Wallace [7].

A vibration neutralizer usually consists of a rigid mass attached to one end of a spring which is attached
to the main structure. The implementation of vibration neutralizers to double-panel structures implies various options of neutralizers attached to the host panels. As for the single panel applications mentioned, vibration neutralizers can be attached to either of the two panels or to both panels. In the latter case, two separated neutralizers are attached to either panels. This paper describes another possible arrangement that the typical vibration neutralizer is extended by a second spring. Thus, one single neutralizer mass can be attached to both panels of the double-panel partition. In fact the two panels are dynamically coupled by the cavity spring and the vibration neutralizer systems. It is intended to improve the low sound transmission loss at the Mass-Air-Mass (MAM) cavity resonance by tuning the resonance of the distributed vibration neutralizers to the MAM resonance frequency.

2 Theory

For the calculation of the sound transmission loss a mechanical model and a beam model are derived.

2.1 Mechanical Model

This investigation has the objective to improve the sound transmission loss of double-panel structures at low frequencies. Thus, as a first approximation the double-panel structure with internal neutralizers can be described as a mechanical system. This simplified approach provides an insight into the general physical principles of the structures of interest. The implementation of vibration neutralizer into the cavity of double-panel structures is shown in Figure 1.

![Figure 1: Mechanical representation of double-panel structure with internal neutralizer.](image)

For the mechanical model all springs are assumed to be massless and have constant complex stiffness $s_i$. Consider the resulting velocities due to a harmonic input force $F_1$ the equations of motion can be written in matrix form as

$$i\omega M \mathbf{v} + \frac{1}{i\omega} \mathbf{K} \mathbf{v} = \mathbf{f}$$  \hspace{1cm} (1)

where the mass matrix $M$ and the stiffness matrix $\mathbf{K}$ are given by

$$M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_r & 0 \\ 0 & 0 & m_2 \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} s_c + s_1 & -s_1 & -s_c \\ -s_1 & s_1 + s_2 & -s_2 \\ -s_c & -s_2 & s_2 + s_c \end{bmatrix}$$

and the velocity $\mathbf{v}$ and force $\mathbf{f}$ vector

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_r \\ v_2 \end{bmatrix}, \quad \mathbf{f} = \begin{bmatrix} F_1 \\ 0 \\ 0 \end{bmatrix}.$$  

The present implementation of vibration neutralizers has the typical intention to increase the resistance of the host structure due to the motion of the neutralizer. The use of vibration neutralizers to suppress vibrational modes of the panels is not the primary objective of this investigation. At resonance the motion of the traditional mechanical vibration neutralizer is out of phase to the motion of the host structure. This results in a low or in the ideal case in a zero motion of the host structure at the resonance frequency of the vibration neutralizer. In the case of a double-panel structure the ideal case at resonance frequency of the vibration neutralizer is high motion of the neutralizer mass and zero motion of the panels. The motion of the panels and the neutralizer mass are out of phase. To find the neutralizer resonance frequency the mechanical system of Figure 1 can be reduced to that shown in Figure 2.

![Figure 2: Mechanical system of Fig. 1 at neutralizer resonance.](image)

With this reduced system the internal vibration neutralizer resonance frequency $\omega_r$ is found to be

$$\omega_r^2 = \frac{s_1 + s_2}{m_r}.$$  \hspace{1cm} (2)

Using Equation (2) the vibration neutralizer can be tuned to a specific frequency, for example the MAM-frequency.

2.2 Beam Model

At low frequencies it is assumed that the internal vibration neutralizers are affected by the modal behaviour of the panels. In order to take the number and position
of distributed vibration neutralizers into account, a one-dimensional beam model was derived.

Following Cremer’s model [8] of floating floors the double-panel partition is described as two panels in bending connected by an elastic layer represented by uncoupled springs. For the one-dimensional case, the two panels of the double-panel partition can be considered as two simply supported beams of length \( l \). This assumption is valid well below the first transverse resonance of the cavity. For an incoming plane wave pressure load \( p_0 \) the beam model is shown in Figure 3.

\[
D_1 \nabla^4 \sum_{n=1}^{\infty} q_{1,n} \Phi_{1,n}(x) - \omega^2 m_1 \nabla^2 \sum_{n=1}^{\infty} q_{1,n} \Phi_{1,n}(x) + s_c \left( \sum_{n=1}^{\infty} q_{1,n} \Phi_{1,n}(x) - \sum_{n=1}^{\infty} q_{2,n} \Phi_{2,n}(x) \right) = p_0
\]

\[
D_2 \nabla^4 \sum_{n=1}^{\infty} q_{2,n} \Phi_{2,n}(x) - \omega^2 m_2 \nabla^2 \sum_{n=1}^{\infty} q_{2,n} \Phi_{2,n}(x) + s_c \left( \sum_{n=1}^{\infty} q_{2,n} \Phi_{2,n}(x) - \sum_{n=1}^{\infty} q_{1,n} \Phi_{1,n}(x) \right) = 0
\]

where \( D_1 \) is the complex bending stiffness; \( p_0 \) is the pressure amplitude of an incident plane wave.

The vibration neutralizers are attached to the structure at discrete positions (Figure 4). The vibration neutralizers can be implemented into the beam model by the coupling point forces \( F_s \) connecting the neutralizer mass to the beams. A single point force \( F_s \) at neutralizer position \( x_s \) is described using generalised coordinates [9] by

\[
F_n = \int_0^l F_s \Phi_n(x) \, dx = \hat{F}_s \Phi_{m}(x_s)
\]

In general, the double-panel system with internal neutralizer described by generalized coordinates is comparable with the mechanical system of Figure 1. This yields for the coupling point forces

\[
F_{1,s}(x_s) = R_1 w_1(x_s) - R_2 w_2(x_s)
\]

\[
F_{2,s}(x_s) = R_3 w_1(x_s) - R_4 w_2(x_s)
\]

In relation to the properties of the vibration neutralizer the introduction of the angular resonance frequencies \( \omega_{1r}^2 = \frac{\omega_1^2}{m_r} \) and \( \omega_{2r}^2 = \frac{\omega_2^2}{m_r} \) leads to

\[
R_1 = \frac{\omega_{1r}^2}{\omega_{1r}^2 - \omega^2} m_r \quad \text{and} \quad R_2 = \frac{\omega_{2r}^2}{\omega_{2r}^2 - \omega^2} m_r
\]

\[
R_4 = \frac{\omega_{2r}^2}{\omega_{2r}^2 - \omega^2} m_r \quad \text{and} \quad R_4 = \frac{\omega_{1r}^2}{\omega_{1r}^2 - \omega^2} m_r
\]

in equation (5). Substituting equations (5)-(6) into equation (4) gives the coupling generalized forces. Multiplying both sides of equation (3) by the modeshape \( \Phi_{m} \) and integrating over \( l \) the coupling generalized forces can be implemented into the beam model. For \( J \) equally tuned vibration neutralizers the resulting equation expressed in matrix form can be written as

\[
(-\omega^2 M + K) q = p
\]

where the stiffness matrix

\[
K = \begin{bmatrix}
K_1 + S_c + R_1 \Phi \Phi^T & -S_c - R_2 \Phi \Phi^T \\
-S_c - R_3 \Phi \Phi^T & K_2 + S_c + R_4 \Phi \Phi^T
\end{bmatrix}
\]

with \( N \times N \)-matrices

\[
K_1 = \frac{l}{2} \begin{bmatrix}
D_1 k_{s,1}^4 & 0 \\
0 & D_2 k_{s,2}^4
\end{bmatrix}
\]

\[
K_2 = \frac{l}{2} \begin{bmatrix}
D_2 k_{s,2}^4 & 0 \\
0 & D_1 k_{s,1}^4
\end{bmatrix}
\]
2.3 Sound reduction index

The sound transmission coefficient $\tau$ describes the ratio of the transmitted sound power to the incident sound power.

$$\tau = \frac{P_{rad}}{P_i}$$  \hspace{1cm} (9)

The sound transmission loss of partitions is defined by the sound reduction index $R$

$$R = 10 \log_{10} \frac{1}{\tau}$$  \hspace{1cm} (10)

or

$$R = 10 \log_{10} \frac{P_i}{P_{rad}}$$  \hspace{1cm} (11)

2.3.1 Incident Power

For this paper only normal sound incidence of a plane sound wave is considered. The incident sound power per unit width is given by

$$P_i = \frac{|\hat{p}_0|^2 l}{2 \rho_0 c_0}$$  \hspace{1cm} (12)

In order to take the reflection at the incident panel into account, the mass matrices of the mechanical and the beam model were modified, see Fahy [10]. For the mechanical model the mass matrix became

$$M = \begin{bmatrix} m_1 - \frac{\rho c_0}{\omega} & 0 & 0 \\ 0 & m_\nu & 0 \\ 0 & 0 & m_2 \end{bmatrix},$$

and the mass matrix of the beam model is given by

$$M = \begin{bmatrix} M_1 - \frac{\rho c_0 l}{2 \omega} & 0 \\ 0 & M_2 \end{bmatrix}.$$  

2.3.2 Radiation for mechanical model

For the rigid mechanical mass $m_2$ representing the radiating panel in Figure 1 the radiated power per unit width is controlled by the spatial mean square velocity $|\hat{v}_2|^2$

$$P_{rad} = \frac{1}{2} \rho_0 c_0 l \langle |\hat{v}_2|^2 \rangle$$  \hspace{1cm} (13)

and assuming a radiation ratio of 1.

2.3.3 Radiation for beam model

For the beam model, the velocity $v_2$ of the radiating panel differs across the length $l$ of the beam. Particularly at low frequencies the radiation behaviour of the natural modes has to be taken into account. For a flexible structure the equation for the radiated power per unit width can be written as

$$P_{rad} = \frac{\sigma \rho_0 c_0}{2} \int_0^l |v(x)|^2 \, dx$$  \hspace{1cm} (14)

where $\sigma$ is the radiation efficiency. Using generalised coordinates, $q_2$ of equation (7) can be substituted into equation (14).

$$P_{rad} = \frac{\rho_0 c_0}{4} \int_0^l |\omega \sum_{n=1}^N q_{2,n}(\phi) \Phi_n(x)|^2 \, dx$$  \hspace{1cm} (15)

The formulation of the modal radiation efficiencies $\sigma_n$ of a beam in flexure were published by Wallace [7]. Along with the orthogonality condition this yields

$$P_{rad} = \frac{\rho_0 c_0 \omega^2 l}{4} \sum_{n=1}^N |q_{2,n}|^2 \sigma_n$$  \hspace{1cm} (16)

3 Parameter study

Using the previous models, parameter studies have been carried out on a double-panel structure represented by 1 mm steel beams with 7.8 kg/m, 40 mm air cavity and 1 m length. The cavity spring was tuned to a MAM-resonance frequency at 250 Hz. Damping in the host structure was assumed to be small with $\eta = 0.01$.

The effect due to the vibration neutralizers was tuned to specific frequencies by equation (2). For all results shown the total mass per unit width of the attached neutralizers had the constant value of 1 kg/m. Figure 5 shows a comparison of the results of the mechanical model with those of the beam model. For this purpose a rather high number of 9 lightly damped ($\eta_r = 0.01$) vibration neutralizers were equally spaced across the
Figure 5: Double-panel structure with internal vibration neutralizers tuned to 160 Hz. (- -) mechanical model; (···) mass law; (—) beam model.

The neutralizers were tuned to a resonance frequency of 160 Hz. For both models the comparison shows the typical behaviour of a vibration neutralizer at its tuned frequency. When a vibration neutralizer is attached to a host structure the peak at the tuned frequency followed by a dip in the vibration characteristic of the host structure always identifies the existence of a vibration neutralizer. Discrepancies between the two models at low frequencies are caused by the more accurate calculation of the radiation of a 1 m beam by the beam model. At higher frequencies the sound transmission calculated by the beam model indicates the influence of the higher order modes. With the conclusion that the two models reveal equal results under the given limitations, the results shown below are calculated only by the beam model.

The present research has the main objective to increase the sound transmission loss at the MAM-resonance frequency. For this reason, the previous arrangement of 9 vibration neutralizers was tuned to the MAM-resonance of the host double-panel structure. The comparison with and without internal neutralizers is shown in figure 6. The calculation of figure 6 has been repeated for damped vibration neutralizers. Figure 7 shows the comparison for the same vibration neutralizer configuration with a damping loss factor of $\eta_r = 0.3$.

The sound transmission loss determined for the structure with damped vibration neutralizers shows the potential of the presented concept. By tuning the vibration neutralizers to the MAM-resonance an increase in Sound Reduction Index of about 10 dB at the MAM-resonance can be achieved. Since only the damping of the neutralizers has been changed, figure 6 and figure 7 differ significantly only at the resonance frequency of the vibration neutralizer. The sound transmission loss at the frequencies above and below 250 Hz are not affected.

For the next steps of this research, it is suggested to attach different tuned vibration neutralizers across the double-panel structure. As a first step to investigate this, a set of calculations are presented in figure 8 in which the tuning frequency of all 9 lightly damped vibration neutralizers is varied across a range around the MAM resonance frequency. It can be seen, that the previous increase in the sound transmission loss caused by the vibration neutralizers can be achieved for frequencies contiguous to the MAM resonance. Thus, by distributing differently tuned vibration neutralizers it is expected to achieve an increase of the sound transmission loss covering a broader frequency range around the MAM resonance.
Figure 8: Results for sets of equally tuned vibration neutralizers tuned to a range of frequencies around MAM resonance

4 Discussion

This paper has analysed the implementation of internal vibration neutralizers in the cavity of double-panel structures. According to the known applications of mechanical vibration neutralizers this paper has emphasized the specific case when the vibration neutralizer is moving out of phase with the panels but not the cancellation or reduction of natural modes of the panels. Two analytical models, the mechanical and beam model, were derived and compared with each other. Under the given limitations of the models, the comparison of both models has revealed good agreement. For initial parameter studies the beam model was used. The results presented have shown the general potential of vibration neutralizers implemented into the cavity of the double panel structure. In particular at the MAM-resonance frequency a promising increase of the sound transmission loss has been determined.

Further investigations of the idea of internal vibration neutralizers should concentrate on finding efficient distributions based on periodic, stochastic or pseudorandom distributions. In this context, the effects due to different tuned vibration neutralizers attached across the structure has to be taken into account. Further work about the concept of internal vibration neutralizers will extend the beam model to oblique sound incidence and present a two-dimensional plate model.

5 Acknowledgements

The authors wishes to thank the German Federal Environmental Foundation for the support for this work by the Doctoral Scholarship Programme. This work was accepted as a research training project within the EU-funded "European Doctorate in Sound and Vibration Studies” programme (EDSVS). With the support of the EDSVS programme parts of this work were carried out at the ISVR in Southampton.

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