Sound propagation in the nocturnal boundary layer

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It is well known that at night there is typically a sound speed inversion creating a sound channel in the lower atmosphere. A modal model which incorporates ground and atmospheric attenuation in a fundamental way has recently been developed. The model will be presented. In addition, some predictions of the modal model and their experimental verification will be described.

![Figure 1: Model sound speed profile](image)

Figure 1: Model sound speed profile

1 Introduction

Sound propagation in the nocturnal boundary layer is characterized by downward refraction near the ground and upward refraction higher up in the atmosphere.[1] A model sound speed profile is depicted in Figure 1. Physically, the downward refraction causes the sound to be ducted along the ground, but the upward refraction causes the duct to be imperfect, allowing sound to leak out of the duct into the upper atmosphere. In addition the propagation is affected by the sound’s interaction with the ground which is both compliant and lossy.

The propagation of a monotone continuous wave of angular frequency $\omega$ is described by solving the Helmholtz equation

$$\left(\nabla_H^2 + \frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c(z)^2}\right)P(x_H, z) = 0$$

(1)

for the acoustic pressure amplitude $P$. Here $x_H$ are the horizontal coordinates and $\nabla_H$ the corresponding Laplacian; $z$ is the height from the ground. The sound’s interaction with the ground is modeled by an impedance $Z$ written in the form

$$Z(\omega) = \frac{i\omega\rho_0}{A(\omega) + iB(\omega)}$$

(2)

so that at the ground one has the impedance boundary condition

$$\frac{\partial P}{\partial z}\bigg|_{z=0} = -(A + iB)P(x_H, 0).$$

(3)

Note that if $B = 0$ then the impedance is pure imaginary and the interaction with the ground does not attenuate the sound field. Various models for $Z(\omega)$ are discussed in [2].

2 The Modal Expansion

A vertical eigenfunction expansion will be used to solve the Helmholtz equation. One considers

$$\left(\frac{d^2}{dz^2} + \frac{\omega^2}{c(z)^2} - \epsilon\right)\psi_\epsilon(z) = 0$$

(4)

with the boundary condition

$$\psi'(0) = -(A + iB)\psi_\epsilon(0).$$

(5)

Then one has the modal expansion [3, 4, 5]

$$P(x_h, z) = \sum_{j=1}^{N} p_{\epsilon_j}(x_h)\psi_{\epsilon_j}(z) + \int_{\Gamma} p(\epsilon, x_h)\psi_\epsilon(z)\,d\epsilon.$$  

(6)

Here $\psi_{\epsilon_j}$ are the square integrable solutions of Equation (4) subject to (5) and $\psi_\epsilon$ are the polynomially bounded solutions. The set of values $\{\epsilon_1, \ldots, \epsilon_N\}$ is called the point spectrum; the $\epsilon_j$ are generally complex valued. $\Gamma = (-\infty, \infty)$ is the continuous spectrum. The functions $p(\epsilon, x_h)$ satisfy the two dimensional Helmholtz equation (with constant coefficients)

$$\left(\nabla_H^2 + \epsilon\right)p(\epsilon, x_H) = 0$$

(7)

subject to appropriate boundary conditions. The $\psi_\epsilon$ are bi-orthogonal[3] and are normalized so that[3, 4]

$$\int_{0}^{\infty} \psi_{\epsilon_j}(z)\psi_{\epsilon_k}(z)\,dz = \delta_{jk}$$

(8)
and

$$\int_0^\infty \psi(z)\psi'(z) \, dz = \delta(\epsilon - \epsilon').$$  (9)

In [3] and [4] techniques were developed to obtain the point spectrum and their corresponding mode functions and to approximate the continuous spectrum. The approach involves solving an auxiliary problem; with the sound speed taken to be constant from its maximum up (so that the duct is no longer leaky, see Figure 2) and $B = 0$ (so that the ground is no longer attenuating); and then systematically controlling the resulting modes as the ground impedance and sound speed are deformed to their physical forms. The result is that, for low angle propagation,

$$P(x_h, z) \approx \sum_{j=1}^{N+M} p_{\epsilon_j}(x_h)\psi_{\epsilon_j}(z).$$  (10)

Here $N + M$ is the number of modes in the point spectrum for the asymptotically constant sound speed profile. One finds [4] that, for $B > 0$, as the sound speed is allowed to become asymptotically upward refracting most of the point spectrum remains point spectrum (as opposed to the lossless case $B = 0$ in which the asymptotically upward refracting model has no point spectrum at all) [6]. There are, however, certain exceptional frequency bands in which some of the eigenvalues in the point spectrum move onto the “unphysical sheet” and cease to be in the spectrum at all. Such modes will be referred to as quasi-modes.

In Figure 3 the spectrum of the asymptotically constant approximation and of the asymptotically upward refracting model are shown at 54 Hz. In this case all of the modes in the asymptotically constant approximation remain modes as the profile becomes asymptotically upward refracting. In Figure 4 the point spectrum is shown at 57 Hz. Here there is one quasi-mode. The contour deformation used to extract the quasi-mode from the continuous spectral integral is shown in Figure 5.

In Figure 6 the corresponding modes, shifted by their phase speed, scaled, superimposed on the sound speed profile, are shown at 54 Hz. In Figure 7 the same is shown at 57 Hz. Note that the single quasi-mode at 57 Hz extends high up into the atmosphere.

In Figures 8 and 9 the accuracy of the approximation 10 is studied. Note that the altitude at which the approximation becomes poor increases with increasing range. For ground to ground propagation the approximation be-
Figure 5: Deformation of the continuous spectra to extract quasi-modes at 57 Hz

Figure 6: Modes at 54 Hz

Figure 7: Modes and one quasi-mode at 57 Hz

Figure 8: Comparing the modal expansion to the full field at 0.5 km

Figure 9: Comparing the modal expansion to the full field at 1.5 km
comes good after 15 to 20 wavelengths from the source. For a source on the ground, the approximation tends to break down, regardless of range, at altitudes of about a third of the turning point height.

3 The Characteristic Properties of the Sound Field

The general properties of low altitude sound propagation in the nocturnal duct can be understood from the dispersion curves for the modes. In general there is a distinguished mode, which we call the surface mode because it becomes the classical surface mode if the sound speed is made to be constant, and a cluster of higher modes.

The surface mode’s magnitude is monotone decreasing with altitude. It attenuates an order of magnitude more rapidly than the other modes and has the slowest group velocity. The other modes all have a sharp first minimum at about the same height. This behavior is a direct consequence of the typical shape of the sound speed profile: it has a steep gradient near the ground and a maximum at an elevation of a few hundred meters. Further, there is no cutoff frequency for the surface mode.

A consequence of the surface mode’s high attenuation rate and the co-location of first minima of the higher modes is the existence of the nocturnal quiet height[7] (see Figure (8)). There is an altitude, a quiet height, at which the sound pressure level is decreased by 10 to 15 dB. This quiet height develops as the surface mode attenuates, is largely independent of range, and is inversely proportional to frequency.

A consequence of the surface mode’s slower group velocity and higher attenuation rate is the universal pulse tail.[8] (see Figure (9)). Since the propagation in the nocturnal duct is modal there is dispersion. As the surface mode propagates more slowly than the rest of the modes it is the last arrival from an impulsive source. In addition, since the attenuation of the surface mode increases with frequency, the high frequency components of the surface mode component of a pulse attenuate rapidly with increasing range. The result is a low frequency tail that is a universal feature of pulse propagation at night.

References


