Empirical Prediction of Fitting Densities in Industrial Workrooms for Ray Tracing

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The objective of this study was to develop an empirical method, based on measurable physical fitting descriptors, from which frequency-dependent fitting densities could be calculated in a workroom for use in the prediction of the sound level in that workroom by ray tracing. The study involved eleven typical workrooms of varying dimensions and types, quantities, and distributions of fittings, in which 125-4000 Hz octave-band sound propagation curves and fitting dimensions had previously been measured. Physical descriptors considered included the Kuttruff fitting density and others involving the number, average height, and total volume of the fittings. Predictions of octave-band sound propagation curves were made for various fitting densities using a ray tracing program. These were compared (based on the total deviation from the predicted curves) with measurements of the sound propagation levels to determine the best fit fitting density. A correlation analysis was then performed between those fitting densities and various physical descriptors to find prediction models. Regression analyses of combinations of the highest correlated parameters found the highest coefficients of determination ranged from 0.39 to 0.61 for octave bands between 125-4000 Hz. The average fitting to workroom height ratio, the volume ratio of the fittings to the workroom, and the number of fittings were the parameters that predicted the fitting density best.

1 Introduction

Noise is an important part of the health and safety of people; in industrial workrooms severe consequences could result when noise levels are too high (i.e. hearing loss and miscommunication). To reduce and minimize noise effectively, it must first be modelled as accurately as possible. Modelling an industrial workroom involves quantifying the fittings (obstacles) by the fitting density. The objective of this study was to develop an empirical method, based on measurable physical fitting descriptors, from which frequency-dependent fitting densities could be calculated in a workroom for use in the prediction of the sound level by ray tracing.

Previous research has found DRAYCUB, a ray tracing approach, to accurately predict sound propagation curves in workrooms [1]. These sound propagation curves, defined as \( SP(r) = L_p(r) - L_w \) [dB], can be predicted for given fitting densities using DRAYCUB. This paper compares DRAYCUB’s sound propagation curves with measurements in workrooms to find which fitting densities best predict the data. Two methods of comparison were used: a slope-fit method, based on the rate of decay of sound propagation levels with distance (the slope of the sound propagation levels); and a best-fit method, based on the total deviation from the predicted curves. Results found better agreement with the best-fit method and it will thus be the focus of this paper.

There has been a great deal of research done on both ray tracing and empirical modelling for predicting sound levels in industrial workrooms [2,3]. Although empirical methods may be fundamentally valid, ray tracing is a more accurate approach [2]. A similar study to the one presented in this paper has been done which used the empirical models in PlantNoise, a model that uses empirical data for modelling, predicting, visualizing and auralising noise in industrial workrooms [4]. Measured results were compared with the sound propagation curves predicted by PlantNoise for fitting densities ranging from 0.025 to 0.15 m\(^{-1}\), and the best-fit fitting densities were correlated with physical descriptors. Simple linear regression was then used with the more highly correlated descriptors (either \( h/H \), the ratio of average fitting height to workroom height; or \( N_f \), the number of fittings) to determine an empirical relationship at octave band frequencies from 125 to 4000 Hz. The R-squared values ranged between 0.25 to 0.47 using this method.

2 Ray Tracing using DRAYCUB

DRAYCUB [5] is a computer program developed from the Ondet and Barbry algorithm [6] which predicts sound propagation curves based on the geometry (size and shape of the room), the fittings (the obstacles in the workrooms, as described by their absorption coefficient and fitting density, \( Q \) [m\(^{-1}\)]), the location and power level of sources(s), the location of receivers, the absorption due to air, and the absorption and diffuse-reflection coefficients of each surface at each octave band frequency. Rays emitted from a source in random directions are traced about the workroom using Monte Carlo methods and vector geometry, based on parameters set by the user (including the number of traced trajectories, the number of rays emitted by the source, and the parameters mentioned above). The
receivers record the number of rays, the energy of each ray, and the total energy of the rays that pass through them. This information is then used to calculate the sound pressure levels at the receivers.

Three assumptions were made while modeling the fittings: they are dimensionless; they scatter omni-directionally; and they are distributed according to a Poisson distribution (if the distance between each pair of fittings were plotted, the frequency distribution of the distances would form a Poisson distribution). The inverse of the mean distance ($\bar{\ell} = 1/Q$) between these pairs, for which the distribution reaches a maximum, represents the fitting density of a given fitted room.

Kuttruff argued that the fitting density is related to the surface area of the fittings. The Kuttruff fitting density, $Q_K$, is defined as the total exposed fitting area divided by four times the volume of the region under consideration [10] as described by $VAQ_{tot} = \frac{Q_K}{4V}$.

Although this relationship does not suggest a frequency-dependence, it is in fact a high-frequency limit. This parameter will be referred to later when determining the fitting densities of the workrooms.

### 3 Workrooms and Modelling

Eleven ‘typical’ workrooms involved in the production of food and personal care products were used in this project (the term ‘typical’ here refers to their size, construction, shape, and fitting distribution). The layout of these workrooms was based on production lines, where machines were often joined by conveyor belts. Plans of the workrooms were used to calculate the dimensions with as much detail as possible (i.e. corners and sloped ceilings). Typical dimensions were 20-30 m on both sides; one was as large as 85 x 50 m$^2$. Typical construction materials included concrete floors, masonry walls and steel deck ceilings.

Although the diffuse-reflection coefficient varies with frequency, sound fields are not highly dependant on it and a value of 0.75 was used for all cases based on previous research [7].

Since it was assumed that all workrooms are of typical construction, an ‘average’ room absorption coefficient was used for each frequency. Based on previous research, the following values were used: 0.15/0.15/0.08/0.07/0.06/0.06, for octave bands ranging from 125 to 4000 Hz, respectively [8]. Although these values can vary by approximately ±25% [9], it was not expected that DRAYCUB predictions would be very sensitive to changes in the absorption coefficients.

The fittings considered, machines and conveyors (the two main obstacles in workrooms), had complex geometries. To determine the physical descriptors related to fittings (i.e. their average height, fitting volume, etc.), the machines and conveyors were considered as either rectangular boxes or vertical cylinders. DRAYCUB allows the effect of the workroom fittings to be taken into account, so the average height of the fittings in a room was also used as the height of the fitted volume. When including the conveyors, this average height was lower than when only using the machines.

As mentioned, the physical descriptors chosen for this study were ones that are easy to quantify and/or measure. They included the Kuttruff fitting density, $(Q_k [m^{-1}])$; the average fitting height, $(h [m])$; the number of fittings, $(N_f)$; the percent of floor area covered by fittings, $(%FAC)$; the total fitting volume to workroom volume, $(V_f/V)$; and the average fitting height to average workroom height $(h/H)$.

These parameters were correlated with the best-fit fitting densities to identify which were most highly related to $Q$. The numerical value for a good correlation depends on the goodness of fit, $r$, for the final data. Correlation was considered weak if $0 \leq |r| < 0.5$ and strong if $0.8 \leq |r| < 1$ [11].

### 4 Workroom Measurements

An omnidirectional loudspeaker array was used to generate broadband noise in the eleven workrooms. The sound pressure levels were then measured in 125-4000 Hz octave bands at increasing distances from the source (i.e. 0.5, 1, 2, 5, 10 m, etc.) and sound-propagation levels were found at each frequency. The paths of sound propagation were chosen through areas of typical fittings. The two fitting cases will be referred to as Machines and Machines and Conveyors for the remainder of this paper. Data for the Machines case was not obtained for one of the eleven workrooms.

### 5 Results and Analysis

DRAYCUB was used to predict sound propagation curves for each workroom at each frequency for varying $Q$-values (0, 0.1, 0.15, 0.2, 0.25, 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8 m$^{-1}$), for receivers in two directions (each with a single source), and for the average fitting height of the Machines and of the Machines and Conveyors configurations. Typical results are illustrated in Figure 1.
Figure 1: Sound propagation curves measured and predicted by DRAYCUB for fitting densities from Q = 0 to 0.80 m⁻¹ for a typical workroom at 1000 Hz. ‘Data’ represents the measurements taken in the workroom.

Although the two graphs in Figure 1 appear similar, upon closer examination, the Machine curves decay slightly faster than the Machines and Conveyors curves. It is also apparent from the measured data points that sound-propagation levels do not follow a constant linear slope as distance increases logarithmically, but rather there is a “break point” near 10 m (keep in mind the x-axis has been plotted logarithmically). As a result, logarithmic regression was used to fit three slopes to the data plotted in Figure 1: one called the ‘full-distance’, that included all data points (from r = 0.5 m to the far end of the room); one called the ‘near distance’ (from r = 0.5 m up to but not including r = 10 m); and one called the ‘far distance’ (from r = 10 m until the far end of the room).

The best-fit Q-values were identified by summing the differences between the measured data points (L_p-L_w) and the sound propagation level for each distance measurement (these were calculated based on the equation for a line found using linear regression in the previous part). In most cases, this sum passed from negative to positive values over the Q-value range; the sound propagation level for each distance was calculated based on the best-fit fitting density was set to 0 m⁻¹. For example, Table 2 illustrates the case where all the values were negative and the best-fit fitting density was set to 0 m⁻¹.

At each frequency, in each room, six best-fit fitting densities were found. As described earlier, the values at each frequency were then correlated with physical descriptors that characterized the corresponding workroom fittings. Only single correlations were possible and thus ratios of some of the physical descriptors listed above were combined and also used in the analysis.

For all but the lowest frequency, the highest correlation values for both the near and far distances were never both higher than that for the full distance (for 125 Hz, the highest correlation values were ‘full’: -0.49; ‘near’: -0.69; ‘far’: -0.52). This suggests that there is no advantage in splitting the distance for the best-fit data (or at least not with the transition at 10 m). The split distances were set aside and the full distance was used for all further calculations.

As shown in Table 3, the highest coefficients of determination for the Machines and the Machines and Conveyors cases were very close. The two cases however didn’t necessarily use the same physical descriptors.

Table 1: Machines and Conveyors data from Figure 1 at 1000 Hz

Table 2: Machine data from the same workroom as in Figure 1, but at 250 Hz

Table 3: Comparison of the best-fit Q-values for the Machines and the Machines and Conveyors cases.
Table 3: Highest adjusted coefficients of determination \( (r^2) \) found for the full distance

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Machines</th>
<th>Machines and Conveyors</th>
</tr>
</thead>
<tbody>
<tr>
<td>125</td>
<td>0.48</td>
<td>0.48</td>
</tr>
<tr>
<td>250</td>
<td>0.61</td>
<td>0.63</td>
</tr>
<tr>
<td>500</td>
<td>0.61</td>
<td>0.63</td>
</tr>
<tr>
<td>1000</td>
<td>0.39</td>
<td>0.33</td>
</tr>
<tr>
<td>2000</td>
<td>0.54</td>
<td>0.4</td>
</tr>
<tr>
<td>4000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the majority of the frequencies, including conveyors did not improve the accuracy (the Machines had a higher coefficient of determination). Unfortunately, it is at midrange frequencies (500 – 1000 Hz) that this is not true. Since only 11 rooms were used to find these data (10 for the Machines), the difference at 1000 Hz may not be that significant. Although using the Machines and Conveyors data is more accurate for the 500 Hz case, the additional time and effort required to measure and include conveyors in calculations may not be worthwhile. For the purposes of this study, since \( r^2 = 0.61 \) at 500 Hz and \( r^2 = 0.60 \) at 1000 Hz are the two highest values for the Machines case, it is assumed that they will predict the fitting densities with sufficient accuracy. Greater accuracy is possible, but it comes at the price of additional field measurements.

The most highly correlated physical descriptors for the best-fit method for Machines using the full distance data were \( h/H \) and \( N_f \).

Using various combinations of the physical parameters (up to three parameters at a time) in multiple-regression analyses, equations were found to predict the fitting density. The equations with the best fit, or highest adjusted \( r^2 \) value based on only one physical descriptor, are listed as Equations (1a –1f):

\[
Q_{125} = 1.27 - 2.97 \left( \frac{h}{H} \right) \quad r^2 = 0.15 \quad (1a)
\]

\[
Q_{250} = 1.19 - 3.07 \left( \frac{h}{H} \right) \quad r^2 = 0.43 \quad (1b)
\]

\[
Q_{500} = 1.21 - 2.50 \left( \frac{h}{H} \right) \quad r^2 = 0.41 \quad (1c)
\]

\[
Q_{1000} = 1.23 - 2.75 \left( \frac{h}{H} \right) \quad r^2 = 0.50 \quad (1d)
\]

\[
Q_{2000} = 1.09 - 2.40 \left( \frac{h}{H} \right) \quad r^2 = 0.39 \quad (1e)
\]

\[
Q_{4000} = 0.428 + 0.00414(N_f) \quad r^2 = 0.48 \quad (1f)
\]

Higher adjusted \( r^2 \) values were obtained with the addition of a second parameter, as listed in Equations (2a-2f):

\[
Q_{125} = 2.13 - 25.7 \left( \frac{V_f}{V} \right) - 3.69 \left( \frac{h}{H} \right) \quad r^2 = 0.48 \quad (2a)
\]

\[
Q_{250} = 1.57 - 11.2 \left( \frac{V_f}{V} \right) - 3.38 \left( \frac{h}{H} \right) \quad r^2 = 0.51 \quad (2b)
\]

\[
Q_{500} = 1.56 - 9.95 \left( \frac{V_f}{V} \right) - 3.02 \left( \frac{h}{H} \right) \quad r^2 = 0.60 \quad (2d)
\]

\[
Q_{1000} = 1.32 - 6.67 \left( \frac{V_f}{V} \right) - 2.59 \left( \frac{h}{H} \right) \quad r^2 = 0.39 \quad (2e)
\]

\[
Q_{2000} = 0.823 + 0.00279(N_f) - 1.12 \left( \frac{h}{H} \right) \quad r^2 = 0.54 \quad (2f)
\]

The \( r^2 \) values at all frequencies are higher with two parameters, except at 2000 Hz, where they are equal. The addition of a third parameter did not increase the \( r^2 \) value further.

Of the 10 workrooms used in this part of the analysis (data for the 11th workroom was only available for Machines and Conveyors), it was suspected that one or more of the rooms may be unlike the others, acting as outliers rather than trend data. To quantify this for the parameters used in Equations 1 and 2 (as well as for \( Q_{125} \) out of interest), the best-fit fitting densities were plotted with the value of each parameter at each frequency. All graphs showed similar results; the graphs for \( V_f/V \) and for \( h/H \) have been included as Figures 2a and 2b. Ideally, the relationship will be linear.

Figure 2a: Scatter plot showing the relation between \( V_f/V \) and the best-fit fitting density for 10 workrooms

Figure 2b. Scatter plot showing the relation between \( h/H \) and the best-fit fitting density for 10 workrooms
Table 4: The standard deviation and some of its properties for each octave band (all are in units of dB)

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Average Deviation</th>
<th>Standard Deviation of the Average Standard Deviation</th>
<th>Maximum Standard Deviation</th>
<th>Minimum Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>1.34</td>
<td>1.19</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>500</td>
<td>1.26</td>
<td>1.18</td>
<td>2.2</td>
<td>0.4</td>
</tr>
<tr>
<td>1000</td>
<td>2.06</td>
<td>1.69</td>
<td>6.2</td>
<td>0.4</td>
</tr>
<tr>
<td>2000</td>
<td>1.96</td>
<td>1.81</td>
<td>4.2</td>
<td>0.4</td>
</tr>
<tr>
<td>4000</td>
<td>1.36</td>
<td>1.31</td>
<td>2.2</td>
<td>0.4</td>
</tr>
</tbody>
</table>

It does not appear from Figures 2a and 2b that the points take the shape of a particularly straight line. As a result, it is difficult to identify outliers. Perhaps with more data (i.e. more workrooms), the general trend would be more apparent and outliers could be removed; for the time being all data was used for the analysis.

There might have been an offset due to errors in the calibration of the equipment that was used to make the measurements in the workrooms. If the standard deviation of the best-fit fitting density was similar at each frequency, it is likely that this was indeed the case. The average standard deviation, its standard deviation, the maximum standard deviation, as well as the minimum standard deviation for each frequency have been tabulated in Table 4. These results will be discussed in further detail in the following section.

6 Discussion of Best-Fit Results

The terms found in Equations 2 were considerably more significant than those found using the slope-fit method: 67% of the terms were significant as compared to 31% from the slope-fit method.

It was expected that the physical parameters used in the equations to predict the fitting density would represent the density and both the vertical and horizontal size and distribution of the fittings. If that assumption were true, physically realistic conclusions could be drawn from Equations (2a-2f), which used h/H, Nf, and Vf/V. Also, the repetition of two of the same parameters in five of the six equations suggests that those parameters may be strongly associated with the fitting density, unlike in the case of the slope fit model where there were different parameters for each frequency. It was surprising however that the Kuttruff fitting density did not correlate well in this work; it does not appear in any of the equations for either the best-fit method, or the slope-fit method.

An interesting result was found at 2000 Hz: it appears as though Equations (1e) and (2e) are equally as accurate (r2 = 0.39 for both). From an engineering perspective, they are equivalent, and Equation (1e) should be used for simplicity. The coefficient of determination for Equation (2e) however is 0.0005 higher, and since Vf/V will already be calculated (it is required for four other frequency bands), the inclusion of this term does not require much additional work.

Because the correlation values were rarely very high (the highest value for the full distance for Machines using the best-fit data was 0.74), there was quite a bit of variance in Figures 2a – 2b. As a result, not only was it difficult to notice any outliers, it was also difficult to see strong trends. More data (i.e. more workrooms) may help distinguish between the two. As it is now, there is no justification for removing any workroom from the analysis.

If there were a constant offset due to the calibration of the equipment, the standard deviation of the total best fit values would be the same for all rooms at a given frequency band. In other words, the standard deviation of the total best-fit values’ standard deviation would be \( \approx 0 \). For example, the total best fit value at 250 Hz for one workroom is –0.9 (as shown in Table 2). The standard deviation is 1.3. These two values exist for each workroom. The standard deviation of the standard deviation listed in the tables similar to Table 2 should be \( \approx 0 \) if there is a constant offset from room to room. This is not the case based on the data from Table 4; the standard deviations of the standard deviations for a given frequency are all above 1 dB. In addition, the distribution within each frequency band is quite wide (all range from ~3.5 to 4.5 dB wide) suggesting that the offset, if there is one, is not sufficiently constant room to room to be distinguished from uncertainties in the actual measurements.

A surprising finding in this work was that many of the correlation coefficients, particularly for the highly correlated parameter h/H, were negative. This is not intuitive; it is expected that the fitting density would increase with an increase in the physical descriptors used in this report. For example, consider the parameter h/H: as the fitting height increases relative to the height of the workroom, the fittings take up more room in the workroom and thus the fitting density should increase. In fact, the opposite took place, with the average correlation coefficient around –0.7. With the exception of Nf, this phenomenon reoccurs with all the parameters in the best-fit analysis. At this time, no explanation is available for this behavior.

Sound propagation modeling is not a precise science; studies have found ray tracing only to be accurate to \pm 2\ dB [12]. Assuming ‘average’ values for the absorption coefficients as well as one value for each of the diffuse-reflection coefficients as explained in the Theory section may in fact be limiting the accuracy of the ray tracing.
7 Conclusions

The sound propagation curves for various fitting densities in 11 workrooms were predicted using DRAYCUB. This data was compared with experimental data, and linear regression was used to compare the curves. Comparisons of the slopes and the best fits of these curves were made for the full, near, and far distance as well as with fittings comprising only machines, and both machines and conveyors. Physical descriptors were then correlated with the fitting densities to obtain equations that could later be used to predict the fitting density in other workrooms. A thorough analysis found that there was no accuracy advantage in including the conveyors in the model, nor did splitting the distance into a near and far section with a transition point of 10 m from the source, or using the slope-fit method, improve accuracy. Equations with coefficients of determination from 0.39 to 0.61 were found using the best-fit approach. The physical descriptors required for the prediction of the fitting density in workrooms are \( V_f/V \), \( h/H \), and \( N_f \). In other words, the effect of fittings can be found from the number and dimensions of the fittings. The results found there to be little variation in the sound-propagation curves close to the source: within approximately 3 m, all the curves are similar. At larger source/receiver distances, the variation was often around 7 dB, large enough that a proper selection for fitting density is necessary.

It may be informative in future work to change the definition of what was considered near and far distances in this study (i.e. vary the transition point from 10 m). If modeling is repeated using DRAYCUB, it might also be informative to use fitting densities larger than 0.8 m\(^{-1}\) since the fitting densities in several workrooms were predicted to be greater than that. It is also recommended that future work look further into why most correlations were negative.

The absolute best physical parameters to describe the fitting density are still unknown; this study provides good results based on a limited number (and a limited number of combinations). More work is required to obtain data that can be compared with all physical descriptors and all combinations thereof. Although the Kuttruff fitting density did not correlate well in this work, it may (perhaps in combination with another (or new) parameter(s)) fit in another relationship quite well. A further analysis on non-linear relationships that combines multiple physical descriptors may also be useful for a better prediction model of the fitting density in workrooms. Future work may also include validation of the equations from this work by using them to predict the fitting densities in other workrooms. For the time being, the equations found in this study provide reasonable accuracy in the prediction of fitting densities in workrooms.

References