Optimization of Vibratory and Acoustical Components of Percussion Instruments: Theoretical and Experimental Results

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This paper presents an overview of our efforts in physical modelling and optimization techniques centred on the shape-optimization of bars and resonators of marimba-type percussion instruments. The general goal of our study was to predict optimal shapes of structural and acoustical components of musical instruments, in order to obtain a given design-set of vibratory and acoustic eigenvalues for each note, within imposed physical and/or geometrical constraints. Finite-element models were implemented and coupled with several optimization techniques, in order to achieve this goal. In our demonstrative computations, simple computational models were used – Timoshenko beam theory for the bars and uni-dimensional wave theory for the resonators – in order to speed-up calculations. We illustrate these issues on several test cases, and then apply the developed techniques to optimize bars and resonators with various spectral signatures. When optimizing to target sets such as typically found on industrial instruments – for instance with frequency-relations of 1:3:9 or 1:4:10 – the optimal shapes obtained were similar to those of common xylophone and marimba bars. More interestingly, optimizations were also performed aiming many other and non-conventional spectral signatures, which produced original shapes and uncommon sounds. For the vibratory and acoustic components studied, it turned out that the simple computational models used (with about one hundred elements) produced results close to those predicted using three-dimensional models with several thousand massive elements, for the 3~6 optimized lower-frequency modes. Furthermore, we constructed prototypes of three different bars and resonators, for experimental validation of the overall procedure, which proved successful.

1 Introduction

As in many engineering fields, musical acoustics and in particular the design of musical instruments may benefit from the power and convenience of physical modelling techniques and optimization tools. Indeed, these have been applied with success, in the recent past and present, by a few researchers, for the optimization of bell shapes [1] and of the bore profiles in wind instruments [2,3]. This paper presents an overview of our efforts in the field, centred on the shape-optimization of bars and resonators of marimba-type percussion instruments. In a previous work, the general goal of our study was to predict optimal shapes of structural and acoustical components of musical instruments, in order to obtain a design set of vibratory and acoustic eigenvalues for each given note, within imposed physical and/or geometrical constraints. Finite element models were implemented and coupled with several optimization techniques, in order to achieve this goal. In our demonstrative computations, simple computational models were used – Timoshenko beam theory for the bars and uni-dimensional wave theory for the resonators – in order to speed-up calculations. However, accepting the increased computational burden, the same optimization techniques can be easily coupled with more refined vibratory and acoustic models.

Apart from the adequacy of the physical models used, two important issues affect the quality of the predictions as well as the computational effort – the optimization technique(s) used and optimized variable space. Here we discuss and illustrate the use of global optimization as well as of local optimization approaches. Global optimization techniques – such as simulated annealing – enable an almost-sure convergence to a global minimum of the objective function to be minimized, provided that the constraints imposed are not unduly restrictive. This is, however, achieved at the cost of a heavy computational effort. On the other hand, local – typically gradient-based – optimization techniques are fast, but prone to get stuck on local minima, producing less-than-adequate results.

Another important issue is the choice of the optimized variables, which condition the size of the searched space during the optimization procedure. For the problem addressed here, this issue relies on the manner used to describe system geometries. Basic physical variables – connected to the finite-element mesh – are convenient, but can only be used on very simple systems, otherwise one may easily end trying to optimize thousands of parameters. For realistic
systems, some reduced-space approach for geometry description must be adopted, in order to cope with rich geometrical features using only a few variables – spline and Bezier functions are two possible such approaches. In this work we used several families of orthogonal functions – trigonometric and Tchebyshev – which proved effective enough.

We illustrate these issues on several test-cases, and then apply the developed techniques to optimize bars and resonators with various spectral signatures. When optimizing to target sets such as typically found on industrial instruments – for instance with frequency-relations of 1:3:9 or 1:4:10 – the optimal shapes obtained were similar to those of common xylophone and marimba bars. More interestingly, optimizations were also performed aiming many other and non-conventional spectral signatures, which produced original shapes and uncommon sounds.

For the vibratory and acoustic components studied, it turned out that the simple computational models used (with about one hundred elements) produced results close to those predicted using three-dimensional models with several thousand solid elements, at least for the 3–6 optimized lower-frequency modes. Furthermore, we constructed prototypes of three different bars and resonators, for experimental validation of the overall procedure. Tests performed on these specimens produced results very close to the design target frequency-relations.

2 Modelling and optimization

2.1 Vibratory components

In our work [4,5,6], to model the bars of variable cross-section dimensions of marimba-like instruments, we used the Timoshenko beam model, which corrects for the effects of rotary inertia and shear deformation [7], when computing the flexural modes. For small vibratory motions, the transverse displacement $y(x,t)$ and slope $\phi(x,t)$ of the free conservative system are formulated as:

$$\rho S(x) \frac{\partial^2 y}{\partial t^2} + k G S(x) \left( \frac{\partial \phi}{\partial x} - \frac{\partial^2 y}{\partial x^2} \right) = 0 \quad (1)$$

$$\rho I(x) \frac{\partial^2 \phi}{\partial t^2} - E I(x) \frac{\partial^2 \phi}{\partial x^2} + k G S(x) \left( \phi - \frac{\partial y}{\partial x} \right) = 0 \quad (2)$$

where the local bar cross-section area and moment of inertia are respectively $S(x) = B H(x)$ and $I(x) = B H(x)^3 / 12$, where $B$ is the bar width, $\rho$ is the specific mass of the bar material, $E$ is the Young modulus, $G = 2\sqrt[2]{(1 + \nu)}$ is the shear modulus and $k$ is a geometric factor for the shear energy. Space-discretization of the system and application of the Galerkin method to (1,2) in terms of suitable flexural shape-functions enables computation of elementary stiffness and inertia matrices, $K_e$ and $M_e$, which after assembling lead to a matrix dynamical equation for the bars:

$$[M_e]\{\dot{Y}(t)\} + [K_e]\{Y(t)\} = 0 \quad (3)$$

from which a classic eigenvalue problem is formulated:

$$\left([K_e] - \omega^2_{bm} [M_e]\right)\{\phi_{bm}\} = \{0\} \quad (4)$$

enabling computation of the system modal frequencies $\omega_{bm}$ and modeshapes $\phi_{bm}(x)$, with modal index $m = 1, 2, \ldots, M$, for each system geometry.

2.2 Acoustical components

We used a simple sound propagation model based on the classic mono-dimensional wave equation for tubes of variable cross-section $S(x) = \pi D(x)^2 / 4$ along their axis, which is described by the Webster equation [8].

$$\frac{\partial^2 p}{\partial t^2} - \frac{c^2}{S(x)} \frac{\partial}{\partial x} \left( S(x) \frac{\partial p}{\partial x} \right) = 0 \quad (5)$$

The continuous systems were discretized in finite conical elements [9], the change of pressure inside the element being described as a polynomial shape function. Again, using the Galerkin method to minimize the residual error of the approximate solution with respect to (5), we obtain a standard matrix dynamical equation for the pressure in the resonators:

$$[M_e]\{\dot{P}(t)\} + [K_e]\{P(t)\} = 0 \quad (6)$$

from which, similarly, acoustical modes are computed as previously described.

2.3 Optimization procedures

Many parameters are involved in a geometry optimization problem, with two undesirable consequences: Firstly, the optimization becomes computationally intensive, and this is further true as the number of parameters to optimize $P_p$ ($p = 1, 2, \cdots$) increases. Secondly, the error hyper-surface $E(P_p)$ where the global minimum is searched will display in general many local minima. In [4] we avoided converging to sub-optimal local minima by using a robust, but greedy, global optimization technique – simulated annealing [10]. In order to improve the computational efficiency, the global optimization algorithm was coupled with a deterministic gradient-
based local optimization technique, to accelerate the final stage of the convergence procedure. Very encouraging results have been obtained, demonstrating the feasibility and robustness of this approach, as well as the potential to address other aspects of musical instrument design. However, a negative side effect was the need for significant computation times, which is ill-suited to the optimization of large-scale systems such as, for instance, carillon bells. Later, we alleviated this problem by significantly reducing the dimension of the search space where optimization is performed [5,6]. This can be achieved in several ways, describing the geometrical profiles of the vibrating components in terms of a limited number of parameters. We chose to develop $S(x)$ in terms of a set of orthogonal functions $\Psi_j(x)$, namely Tchebyshev polynomials and trigonometric functions, and performing the optimization on their amplitude coefficients. For complex systems, computed using finite-element meshes with hundreds or thousands of elements, this approach reduces the size of the optimization problem by several orders of magnitude. Furthermore, we found that most often acceptable solutions are obtained using efficient local optimization methods, leading to a further reduction in computation times.

In any optimization problem the objective is to find the values of a set of variables which maximizes or minimizes a given error function, usually satisfying a several imposed restrictions. In the present case, we find the optimal shapes of bars and resonators, described in terms of their variable cross-section $S(x)$ and length $L$, which minimize the deviations of the computed modal frequencies $\omega_m[S(x), L]$ from a given reference target-set $\omega_m^{ref}$. The error function was formulated as:

$$E[S(x), L] = \sum_{m=1}^{M} W_m \left( 1 - \frac{\omega_m[S(x), L]}{\omega_m^{ref}} \right)^2$$  \hspace{1cm} (7)

where the $W_m$ are weighting factors for the relative errors in modal frequencies and $M$ is the number of optimized modes. The optimization was performed subjected to geometrical restriction on the system cross-sections ($H_{min} \leq H(x) \leq H_{max}$ for the bars and $D_{min} \leq D(x) \leq D_{max}$ for the resonators) and length ($L_{min} \leq L \leq L_{max}$). Details of our algorithmic implementations are discussed in [4-6].

### 3 Numerical Results

Lack of space prevent us, unfortunately, from presenting here more than a few representative optimized bars and resonators. Figure 1 shows the convergence of the optimization procedure for a (quite untypical) vibraphone bar, as the number of shape-describing functions is increased. The increasingly complex bar shapes are shown on the left column and the corresponding modal frequencies on the right.

![Figure 1 – Optimization of a vibraphone bar, tuned with frequency-ratios 1:2:4:8:16, using trigonometric functions as shape-descriptors.](image)

![Figure 2 – Optimization of a vibraphone resonator, tuned with frequency-ratios 1:3:5:8, using Tchebyshev polynomials as shape-descriptors.](image)
A similar convergence behavior is shown in Figure 2, for a resonator with partials tuned at frequency-ratios 1:3:5:8. In contrast with the previous example, where trigonometric functions were used as shape descriptors, here Tchebyshev polynomials were adopted.

Figure 3 – Two optimizations of a vibraphone resonator, tuned with frequency-ratios 1:3:8:12, using either (a) Trigonometric or (b) Tchebyshev functions.

Figure 4 – Two optimizations of a vibraphone resonator, tuned with frequency-ratios 1:3:5:8, using either (a) Trigonometric or (b) Tchebyshev functions.

Notice that, as illustrated in Figures 3 and 4, the use of different shape-describing functions may produce quite dissimilar optimized shapes as well as rather similar ones. Indeed, it is well known that the mathematical problem of finding shapes that produce a given set of eigenvalues is not univocal. Furthermore, the optimized frequencies are certainly identical, but all other modes will be in general different. In any case, even for identical frequencies, different geometries will obviously produce different modeshapes.

As a last example of optimized results, Figure 5 presents a number of bars designed for very different target-sets of modal frequency-ratios, all of them very un-orthodox. All these bars are tuned to the same fundamental pitch. However, they obviously will display sound characteristics which are quite different from those of conventional instruments. Our dynamical time-domain simulations, based on physical modeling of mallet-excited bars, highlight many interesting features [5,6]. The potential of optimization-based instrument research, when seeking new timbral qualities becomes then obvious.

Figure 5 – Optimizations of several non-orthodox vibraphone bars using trigonometric shape-describing functions.

4 Experimental validation

In order to fully validate the approach developed in the present work, we built and tested 3 different bars and tuned resonators, tuned to 285 Hz, with their shapes optimized so that they should display the following frequency-ratios: 1:1.67:4:10, 1:4:10 and 1:4:6.67:10. Figure 6 displays these prototypes. The aluminium bars were machined in discrete steps, with considerable precision. Resonators were built by piling up Plexiglas boards with 7.74 mm thickness, in which conical holes were machined. As a side point, it is interesting to notice that the optimized 1:4:10 resonator is
substantially smaller than a conventional closed-open resonator tuned to the same frequency (180 instead of 290 mm). This fact, often noticed in optimized resonators, may be of practical significance.

Figure 6 – Optimized tested prototype bars and resonators. From left to right: partials tuned with frequency-ratios 1:1.67:4:10, 1:4:10 and 1:4:6.67:10.

Figure 7 – Impact testing for the modal identification of the optimized bars.

In Figures 7 and 8 we show the simple experimental setups used for identification of the vibratory and acoustic modes. Impact tests and white-noise signals were used, respectively, to obtain the system transfer functions. Modal identification was achieved using a MDOF frequency-domain fitting method. Results are shown in Figures 9 and 10.

Figure 8 – Experimental setup for the modal identification of the optimized resonators.

Figure 9 – Measured transfer functions of the three optimized prototype bars, indicating the target frequency-ratios and the experimentally identified values.

From these results it is clear that the overall objective of this research – being able to design shapes which produce target frequency-ratios of several optimized partials – was indeed achieved. This shows the adequacy of the modeling and optimization techniques used in this work. However, concerning the absolute values of the modal frequencies, our results also showed that, if the physical properties of the materials used (namely $\rho$ and $E$) are not carefully measured...
prior to computations, then all modal frequencies may be translated from the target values, producing less-than-acceptable tuning errors. This aspect, which may be properly accounted for homogeneous well-controlled materials, becomes more problematic when dealing with materials such as wood.

Figure 10 – Measured transfer functions of the three optimized prototype resonators, indicating the target frequency-ratios and the experimentally identified values.

5 Conclusions

This paper reviewed techniques we have been using in recent years for the shape-optimization of musical instrument components. To illustrate our methodology we chose, for cost and convenience, the mallet percussion family. However such methods are obviously applicable to any sort of instrument. Typically, the optimization results presented were obtained after just a few minutes of computation. Use of global optimization methods, however, increases the computational burden by more than one order of magnitude.

We believe that optimization techniques will be increasingly used in the future, not only to improve tuning, but also to create novel instruments and timbres. Other applications of optimization on musical instruments will be undoubtedly studied, for instance to improve radiation of soundboards, to design complex bridges and optimize their location, or for optimal implant of wind instrument tone-holes. Most of these applications will obviously ask for very accurate physical 3-D models of the addressed phenomena. This means brute-force lengthy computations – which is still quite acceptable, if facing the traditional alternative of building costly prototypes one after another.

References