Sustainable Management of an Hake–Anchovy Peruvian Ecosystem Model by Viability Methods

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Figure: Comparison of observed and simulated biomasses of anchovy and hake using a Lotka–Volterra model with density-dependence in the prey. Model parameters are $R = 2.24$, $L = 0.98$, $\kappa = 64672 \times 10^3$ t ($K = 35800 \times 10^3$ t), $\alpha = 1.230 \times 10^{-6} \; t^{-1}$, $\beta = 4.326 \times 10^{-8} \; t^{-1}$. 
### Conservation and catch thresholds

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Lotka–Volterra Model With Density–Dependence

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\begin{align*}
  y(t + 1) &= y(t) \left( R - \frac{R}{\kappa} y(t) - \alpha z(t) - v(t) \right), \\
  z(t + 1) &= z(t) \left( L + \beta y(t) - v(t) \right),
\end{align*}
\]

- state vector \((y, z)\) represents biomasses,
  - \(y\) prey biomass: anchovy
  - \(z\) predator biomass: hake
- control vector \((v, w)\) is fishing effort of each species,
- catches are \(vy\) and \(wz\) (measured in biomass),
- \(R_y\) and \(R_z\) are annual growth rates.
The Lotka–Volterra Model With Density–Dependence is given by the following equations:

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The **viability kernel** is the set of initial states \((y(t_0), z(t_0))\) from which can emerge a trajectory \((y(t), z(t))\), \(t = t_0, t_0 + 1, \ldots\) driven by appropriate controls \((v(t), w(t))\), \(t = t_0, t_0 + 1, \ldots\) such that the following goals are satisfied

- **preservation** (minimal biomass thresholds)

\[
y(t) \geq y^b, \quad z(t) \geq z^b
\]

- and **conservation** requirements (minimal catch thresholds)

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v(t)y(t) \geq Y^b, \quad w(t)z(t) \geq Z^b.
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Figure: The state constraint set is the large set. It includes the smaller viability kernel.
Explicit expression for the viability kernel

Proposition

- If the growth rates are decreasing in the fishing effort
- and if the thresholds are such that the following growth rates are greater than one

\[ R_y(y^b, z^b, \frac{Y^b}{y^b}) \geq 1 \quad \text{and} \quad R_z(y^b, z^b, \frac{Z^b}{z^b}) \geq 1, \]

the viability kernel is given by

\[ \{(y, z) \mid y \geq y^b, \quad z \geq z^b, \quad yR_y(y, z, \frac{Y^b}{y}) \geq y^b, \quad zR_z(y, z, \frac{Z^b}{z}) \geq z^b\}. \]
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Adjusting Catches to Prominent Biomass Conservation Thresholds

1. Considering that first are given minimal biomass conservation thresholds

\[ y^b \geq 0, \quad z^b \geq 0 \]

2. the following catches levels are susceptible to be sustainably maintained:

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\begin{align*}
Y^{b,*} & := y^b \max \{ v \geq 0 \mid R_y(y^b, z^b, v) \geq 1 \} \\
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Hake–anchovy Peruvian fishery: official quotas and sustainable quotas given by the viability approach

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<td>Hake</td>
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E. Ocaña, M. De Lara, R. Oliveros–Ramos and J. Tam
PSI, Tahiti, 02-06 March 2009
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- Contribution to avoid confusion between
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- Managing ecological and economic conflicting objectives
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